Chords & Calculators

The Metaphor of Math and Music Theory

by
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Introduction

The trees are important, but let’s look at the forest for a bit. Let’s look beyond the black spots on the page and see what else the music holds in store. In my 37 years as a music theory teacher, I have often asked my students to step back from the details and look at the bigger picture. I believe they need a broader perspective, a more holistic view. They need to explore the vast forest to really understand the significance of the trees.

Pythagoras, the Greek philosopher and mathematician, had an interesting take on this notion. He once said, “There is geometry in the humming of the strings. There is music in the spacing of the spheres.” If you are at all familiar with his work and accomplishments, you will know that he certainly had a broad perspective. He envisioned the big picture.

Making connections was something Pythagoras did. To me, music theory has always been about making connections. Going from one note to the next, connecting the linear movement of pitches is what we call voice-leading. How pitches are connected vertically is known as harmonic structure. Moving from one chord to another, connecting one harmony to the next is what we refer to as harmonic progression. We study the connections between the horizontal and the vertical, the relationships of recurring patterns, the associations within larger formal structures, and a multitude of other interconnected aspects of music.

The 19th-century physicist and physician, Hermann von Helmholtz, commented on the connections he perceived between math and music when he said, “Mathematics and music, …supporting each other, as if to show forth the secret connection which ties together all the activities of our mind…”[1] This music and math connection has even been noted by the popular singer-songwriter, James Taylor. He once said in a magazine interview, “Music is true. An octave is a mathematical reality.”[2]

We have all heard the phrase, “thinking outside the box.” Helmholtz was certainly an example of that kind of thinking, exploring beyond the established parameters. The idea of “the box” implies that something is contained or isolated and other things are excluded. I would suggest that we not only think outside our boxes but that we take our various boxes and overlap them. Let's become inclusive and not exclusively self-contained. Let’s take the music theory box, which is typically isolated from other areas of study, and overlap it with as many diverse boxes as possible. In this study, math is the other box.

Albert Einstein was certainly a man with overlapping interests. He once said, “If I were not a physicist, I would probably be a musician. I often think in music. I live my daydreams in music. I see my life in terms of music.” This last concept could be considered metaphor. In his book, A Whole New Mind, Daniel Pink puts forth the notion that metaphor is simply “understanding one thing in terms of something else…”[3]

That’s what this book is, a metaphor of math and music theory, understanding one in terms of the other. It’s a tool for seeing a bigger picture, for making connections, and overlapping our boxes. All of these should help us grasp some basic concepts of both music theory and math in new, unique, and insightful ways. It’s my attempt to explore both the forest and the trees.

About This Book

This book assumes very little on the part of the reader/student. The only expectation is an understanding of basic music notation. The reader/student should already know how to read notes in both the treble and bass clefs, as well as ledger lines above and below each. They should also understand the functions of sharps, flats, double sharps and flats, and naturals.

A conversational approach is how the text is presented. The verbalization of each concept is written as if I am interacting with a room full of students or maybe a single student. I want the reader/student to feel we are exploring and discovering new ideas together, not that someone is lecturing from a position of authority or superior knowledge. I designed this to be straightforward, easy to read, and understandable for new music theory students. It is not written to impress other scholars.

In the text itself, I have incorporated various fonts and styles in an attempt to direct the reader's attention to the various concepts we are exploring. All *musical terms* are italicized. Mathematical equations are in a different font so they will stand out from the conversational text. Musical symbols are in the same font as the math equations because they overlap and are interconnected. Since this study is under the umbrella of metaphor, each verbal metaphor that is used is in green. I hope that including these will add a bit of humor and draw attention to this form of symbolism and the conceptual overlapping of ideas.

Throughout the book there are several words or phrases that are underlined and in blue. Each of these is a hyperlink to either an external website or an internal bookmark. Of course, these will not function in a printed version of this book, but are intended to facilitate cross-referencing in the electronic book form. Also included for the e-book format are audio clips and videos that serve as supplemental resources intended to facilitate comprehension.

Incorporated in the text is an abundance of graphics. The most common will be illustrations displayed on a music staff, a piano keyboard, and a circular clock-type graphic. There are also mathematical charts, graphs, and diagrams. Quite a few of these examples include color coded elements. Explanations for some of the color variations will be included in the text as they are introduced. Other colors are added simply to differentiate one example from another.

One other graphic element will be seen throughout the book. It is a red set of two beamed eighth notes (♫). These are included to simply bring the reader's attention to something of significance that might otherwise be overlooked in the midst of the conversation.

All of these various fonts, graphics, and color-coded elements are to give the student multiple opportunities for visualizing and sorting out the concepts being discussed. They are integral to the study. They should be considered significant and utilized in every presentation of the material.

Words, pictures, colors, humor… I hope all of these come together to make math and music theory a bit easier to digest.

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Appendices

Section 1

The Musical Alphabet and Modulo 7

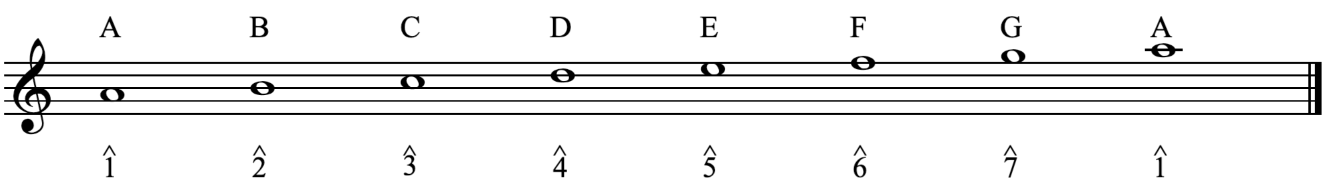
Music uses an “alphabet” with only 7 letters (A B C D E F G). When the sequence reaches G it begins again with A. (A B C D E F G A B C D E…)

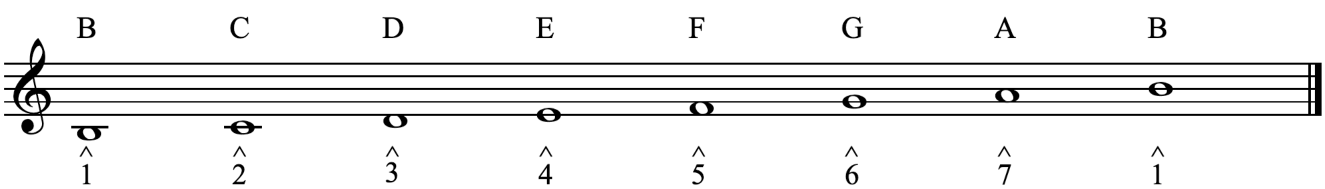
A *(diatonic) scale* in music is a sequence of *pitches* (with corresponding letter names) that encompasses all seven of the letters of the musical “alphabet.” A (*diatonic*) *scale* can begin on any of the seven *pitches* (letters) and progress through the remaining *pitches*.

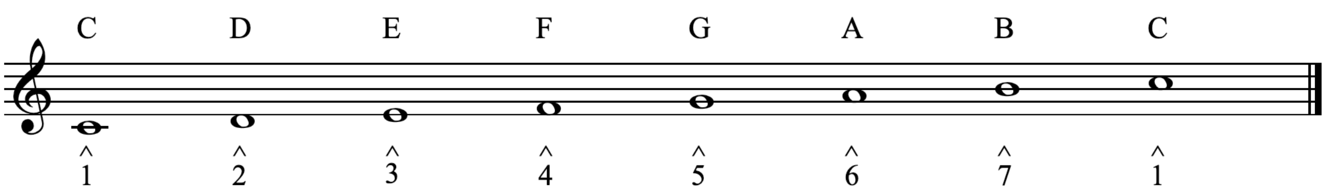
examples: C D E F G A B (C); F G A B C D E (F); etc.

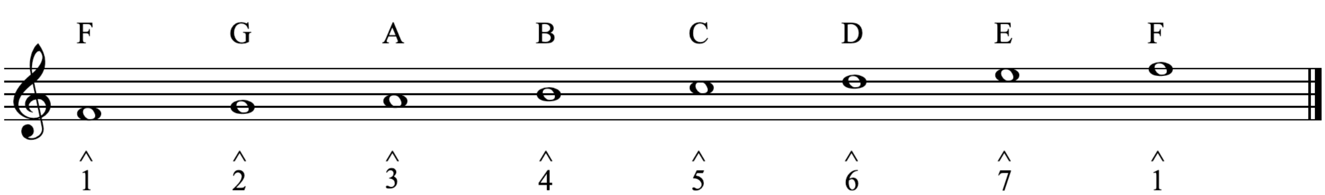
When numbering the *pitches* of a (*diatonic*) *scale*, $\hat{1}$ will correspond to the *pitch* on which the *scale* is based. This *pitch* (or *tone*) is referred to as the *tonic*. The numbers are then referred to as *scale degrees*. A *scale degree* is designated as a number with a circumflex (^) above.

examples:









♫ Modulo 7 is a system of numbering, using only 1 through 7. Any given number larger than 7 will be divided by 7 with the remainder being its congruent number.

The symbol for congruent is ≡

examples: $10 \div 7 = 1$ with a remainder of 3, so $10 \equiv 3$ (modulo 7)
$16 \div 7 = 2$ with a remainder of 2, so $16 \equiv 2$ (modulo 7)

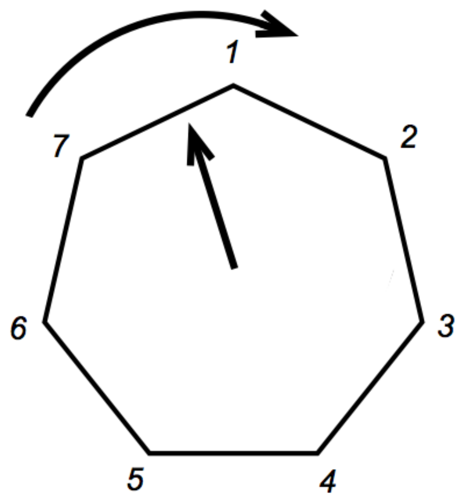
As long as the given number is larger than 7 and smaller than 15, a simpler way to figure the congruent number is to subtract 7 from the given number.

example: $10 - 7 = 3$, so $10 \equiv 3$ (mod 7)

This system simply involves counting from 1 to 7 then starting over at 1 as you continue.

							1	2	3	4	5	6	7	1
1	2	3	4	5	6	7	(8	9	10	11	12	13	14	15…)

It would be like using a 7-point "clock." Each time the hand passes 7 it would move on to 1 and begin again.



♫ In music, mod 7 is used in conjunction with *diatonic* structures. The relationships of the numbers are relative to the specific *tonic pitch* and corresponding *scale*, as seen in the musical examples on the previous page. The *tonic pitch* will always be designated as $\hat{1}$.

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Section 2

The Chromatic Pitch Set and Modulo 12

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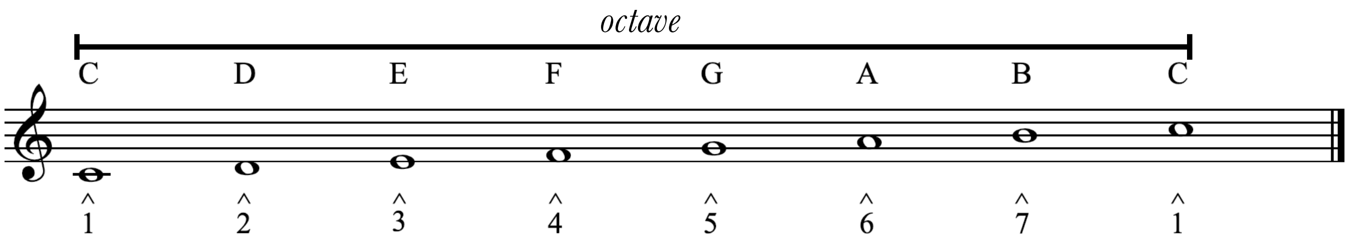
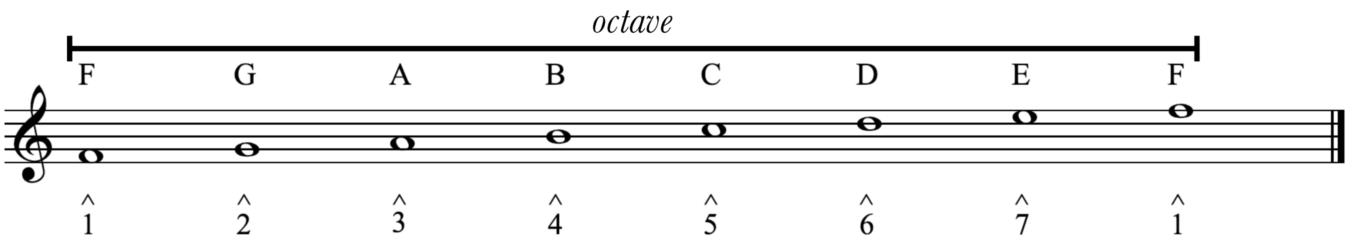
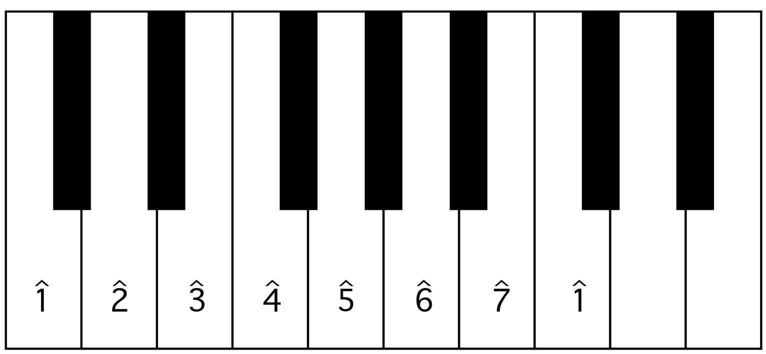
When a (*diatonic*) *scale* is utilized, there are seven unique *pitches* that correspond to the seven letters in our musical alphabet. When the *scale* is constructed, beginning with the *tonic* pitch ($\hat{1}$) and progressing through the seventh *scale degree* ($\hat{7}$), the *tonic* is usually repeated at the end of the sequence to complete the *scale*. This fully constructed *scale*, beginning and ending with the *tonic* pitch, encompasses what is referred to as an *octave*.

examples:

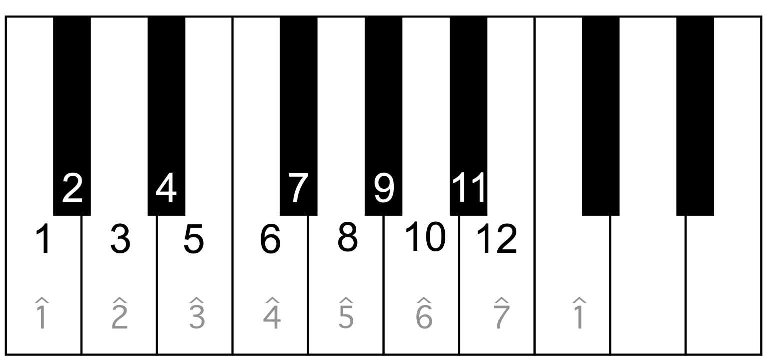




The *scale* above that begins and ends with C, when seen on a piano keyboard, would be represented like this.



Notice that the C *scale* only utilizes seven white keys on the keyboard (eight, if you count the repeated C at the *octave*). If you count both the white and black keys from $\hat{1}$ to $\hat{7}$, there are actually 12 different keys/*pitches*, as seen here.



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This is what we call a *chromatic scale*. It includes every *pitch* within one *octave*.

The distance between each of the *pitches* in this *scale* is referred to as a *half-step*. On a keyboard, that's the distance between two adjacent keys/*pitches* with no other keys/*pitches* in between.

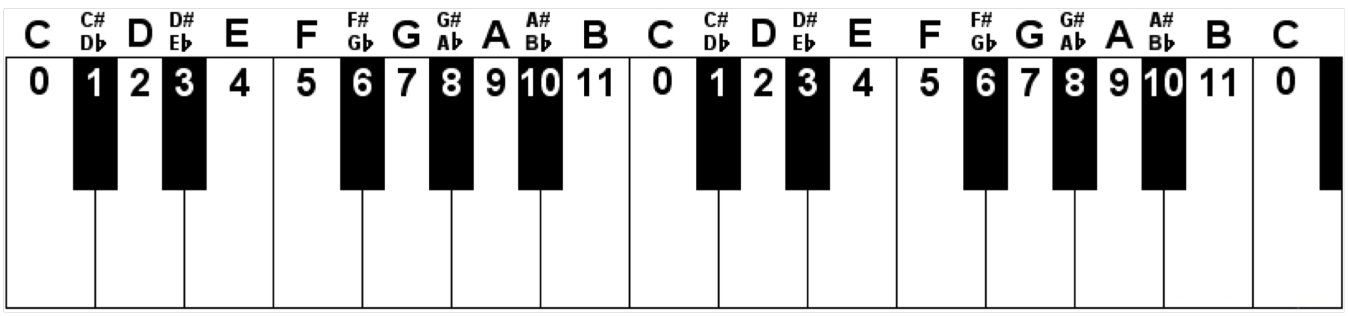
Since our musical alphabet only has seven letters, which correspond to only the white keys on the keyboard, there has to be a way to designate and label the black keys.

♫ Any *pitch* within our musical alphabet can be raised by one *half-step* by adding a sharp (♯).

♫ Any *pitch* from our alphabet can be lowered by one *half-step* by adding a flat (♭).

These designations can be seen on the keyboard below. Notice that each black key has two different designations, one coming from below (the left), by adding a *sharp* (♯), and one coming from above (the right), by adding a *flat* (♭). You see that C♯ and D♭ reside on the same black key. F♯ and G♭ share the same key, and so on.

♫ These are called *enharmonic pitches*. They sound the same when played on the keyboard, but have different names. They will also have different *functions* in various contexts.



You're probably wondering… "What about E & F and B & C? There are no black notes between those keys." You can certainly have an E♯. It would be *enharmonically* the same as an F. You can also have an F♭. It's *enharmonically* the same as an E. This would also be true for B & C.

I'm sure you've noticed that each of the keys on this keyboard are numbered. Those numbers will be used in the related math concepts we will be considering throughout this study. They are the basis for the Modulo 12 system.

• • •

♫ Modulo 12 is a system of numbering, using 0 through 11. Any given number larger than 11 will be divided by 12 with the remainder being its "congruent" number.

example: $16 \div 12 = 1$ with a remainder of 4, so $16 \equiv 4 \pmod{12}$

As long as the given number is larger than 11 and smaller than 24, a simpler way to figure the congruent number is to subtract 12 from the given number.

example: $16 - 12 = 4$, so $16 \equiv 4 \pmod{12}$

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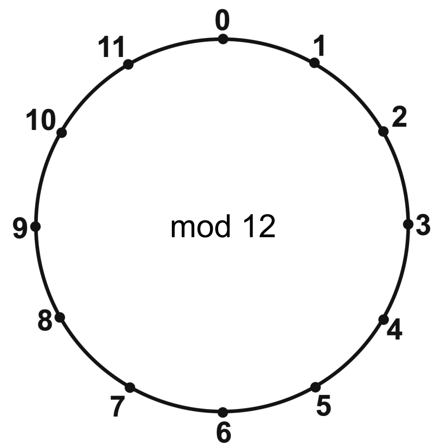
This system simply involves counting from 0 to 11 then starting over at 0 as you continue.

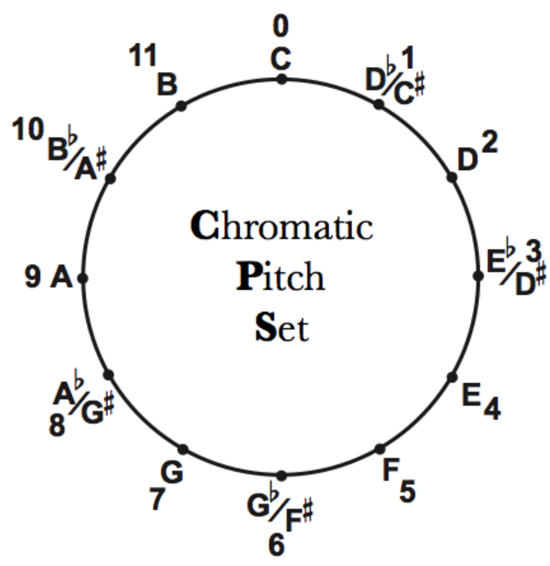
0 1 2 3 4 5 6 7 8 9 10 11 0…
0 1 2 3 4 5 6 7 8 9 10 11 (12 13 14 15 16 17 18 19 20 21 22 23 24…)

The mod 12 "clock" is laid out like a regular clock with one exception, instead of using the number 12, 0 is used.



♫ Also, unlike the mod 7 "clock" on which the numbers are relative to a specific *diatonic scale*, the numbers on the mod 12 "clock" always correspond to a given set of specific *pitches*.

In this study, as in most settings of musical *set theory*, the *pitches* and corresponding numbers of mod 12 will always be fixed. 0 will always be C. 1 will always be C♯/D♭, and so on. The relationships of these numbers will not change according to any *diatonic* structure that is extrapolated from the **C**hromatic **P**itch **S**et (**CPS**).



The labeled keyboard and CPS graphics are available at the end of the book for your reference.

Section 3

Intervals Ascending

♫ A musical *interval* is the vertical (*harmonic*) distance between two *pitches*. The musical terminology for it includes both the quality of the *interval* and the actual distance (how far away one *pitch* is from the other).

The quality is designated by a single letter. Here are the letters and their designations:
P = perfect M = major m = minor d = diminished A = augmented

The distance values are:
U = unison 2 = second (2nd) 3 = third (3rd) 4 = fourth (4th) 5 = fifth (5th)
6 = sixth (6th) 7 = seventh (7th) 8 = octave

Here are examples of how interval designations are written:

examples: m2 = minor 2nd (quality, distance) P4 = perfect 4th (quality, distance) M6 = major 6th (quality, distance)

Let's look at the different types of *intervals* from a numeric, objective, mathematical perspective. The following is a chart of *intervals* with the number of *half-steps* between the two *pitches*. As examples, we'll add some graphic representations for a few of them on a *staff*, a keyboard, and our CPS (from Section 2).

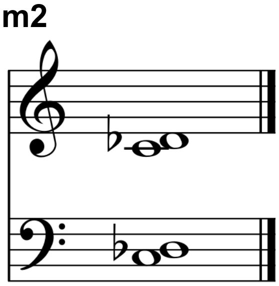
LABEL	PU	m2	M2	m3	M3	P4	A4 d5	P5	m6	M6	m7	M7	P8
# of half-steps	0	1	2	3	4	5	6	7	8	9	10	11	12

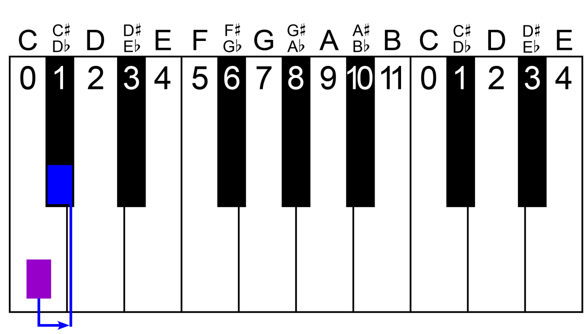
If you noticed the title of this section, it referred to "ascending" *intervals*. This simply means we'll be considering the relationship of the two *pitches* from the lower one to the one above. We'll discuss "descending" *intervals* in the next section.

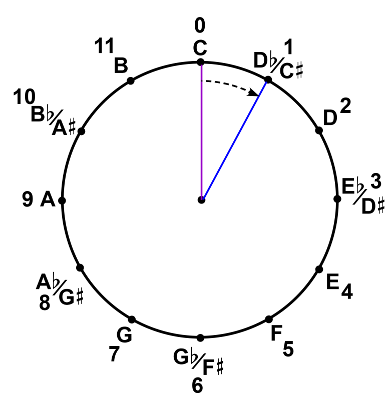
At this point in our study, we'll base our intervals on C as the lower *pitch*. This will allow us to use the CPS and our labeled keyboard as easy references.

Let's look at four of the *intervals* noted above in three different graphical forms.

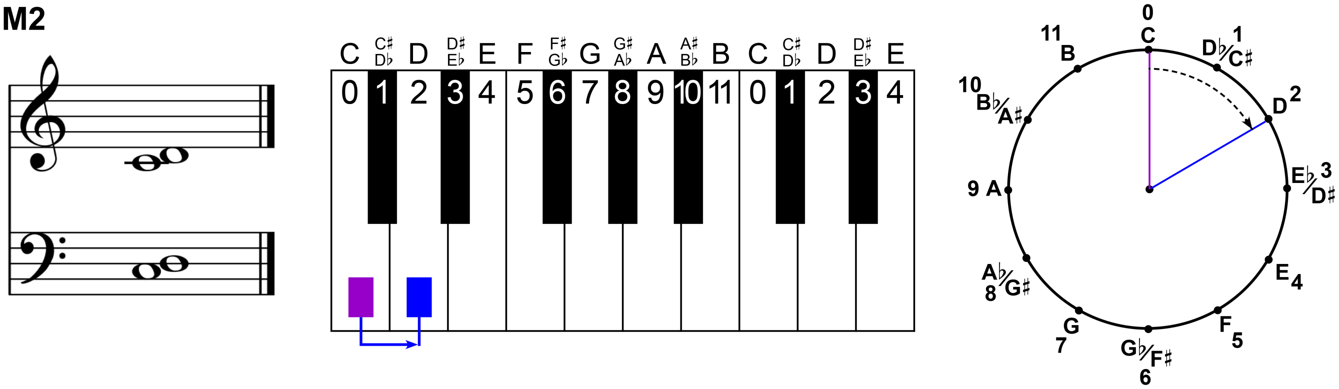
m2



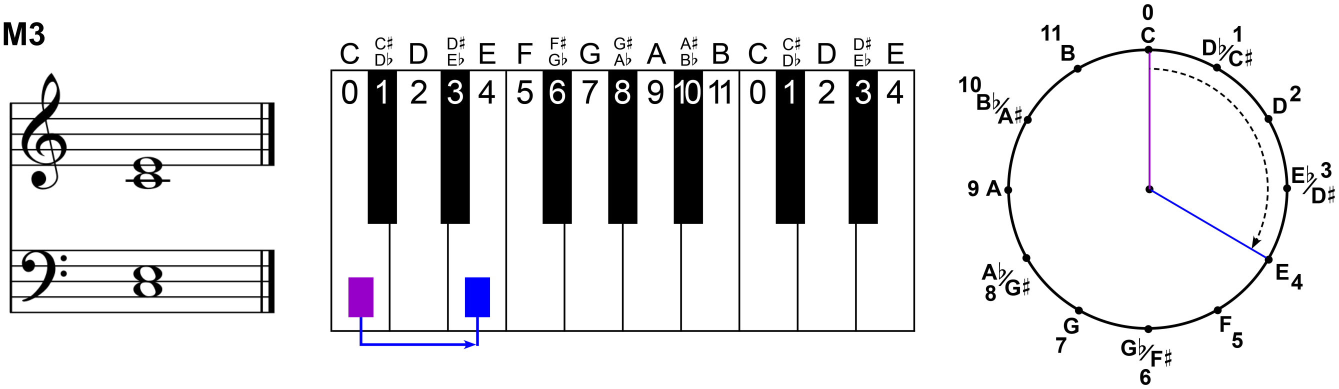




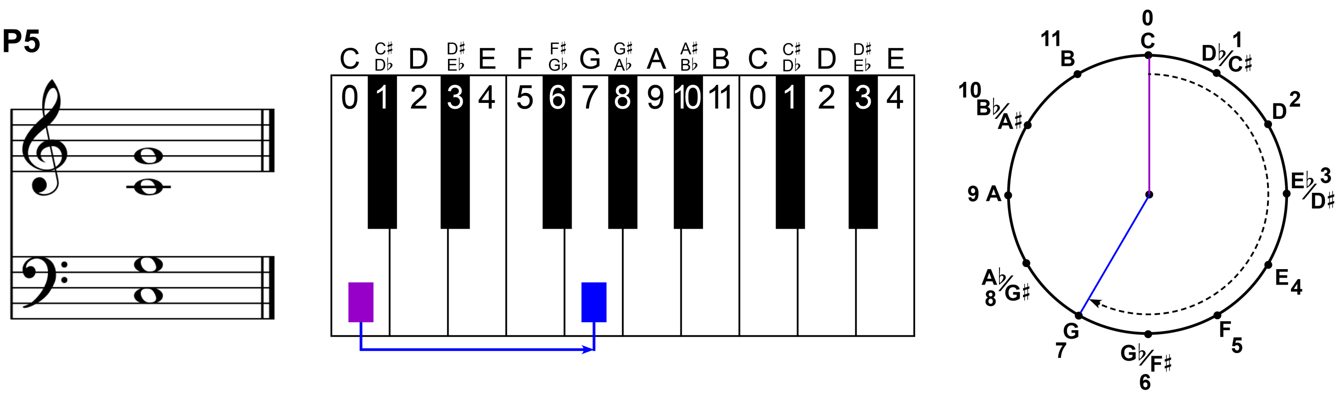
M2



M3



P5

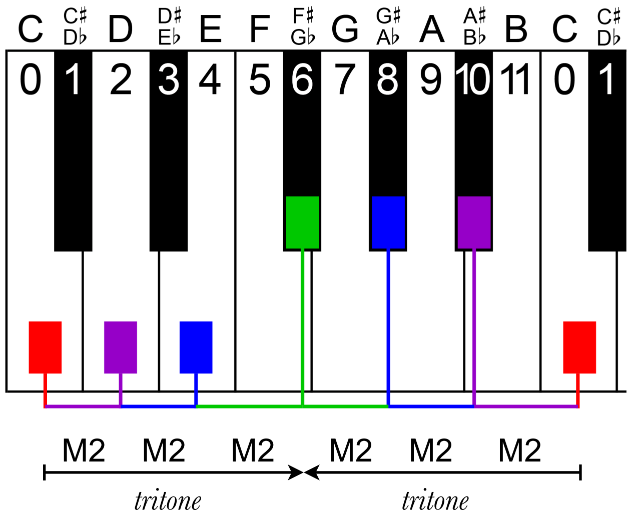


♬ We should probably make note at this point that a M2 (made up of 2 *half-steps*) is often referred to as a *whole-step* or a *whole-tone*. This little tidbit of information will come in handy as we consider the next paragraph or two.

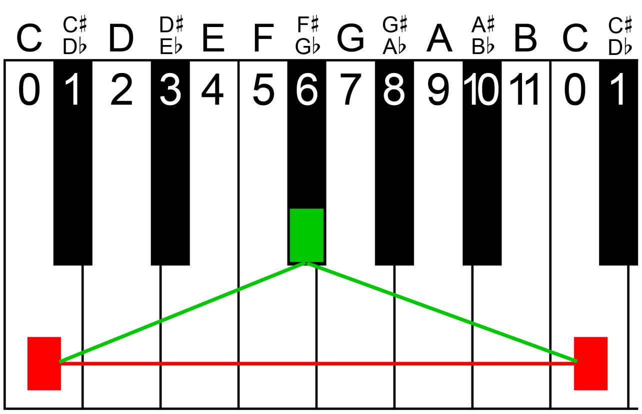
If you noticed in the *interval* chart on the previous page, one of the *intervals* was given two designations. The *interval* that has 6 *half-steps* was labeled as an *augmented fourth* (A4) and a *diminished fifth* (d5). The A4 would be from C up to F♯. The d5 would be from C up to G♭. Those are *enharmonically* the same but will serve different *functions* in different contexts (something we'll cover later). Often, instead of giving this *interval* the typical quality/distance designation, it is referred to as a *tritone*… and it has some colorful mathematical features.

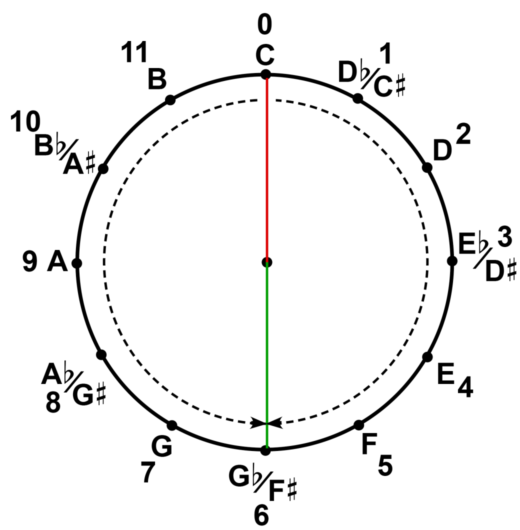
If you take the 6 *half-steps* that form the *tritone* and divide them into 3 equal groups, you get 3 *whole-tones.* So, 3 *whole-tones* equals 1 *tritone*. On our keyboard below, the 3 *ascending whole-tones* are C to D, D to E, and E to F♯.

You should also note that three *descending whole-tones* will result in the same *interval*. These 3 *whole-tones*, C to B♭, B♭ to A♭, and A♭ to G♭, will give you a *tritone*.



There's one more interesting fact to consider before we move on. The *tritone* is the exact center of the *octave*. The keyboard and CPS illustrations below should help us visualize this.





Of course, you must realize that we can't just build *intervals* around the pitch C. Any *interval* can be built from any *pitch*. The starting *pitch* may be different, but the *half-step* relationships will be exactly the same.

Let's use a math equation to help us make the shift away from C. This equation will work no matter what *pitch* you start with. Let's use the letter p to serve as our variable. The letter p will represent the *pitch* we use as the starting point from which we measure the distance to the related *pitch*. p can be any *pitch*/number from our **C**hromatic **P**itch **S**et.

♫ In mathematical terms, a musical *interval* is simply a set of two *pitches*/numbers. A set that represents an *ascending perfect fifth* (P5), starting from C, would be {0,7}. From our CPS, 0 is C and 7 is G. This can be written as:

P5 = {0,7} (mod 12)

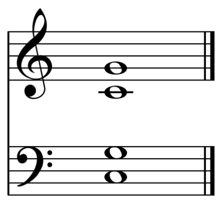
The designation (mod 12) is used to indicate that the Modulo 12 numbering system is being used.

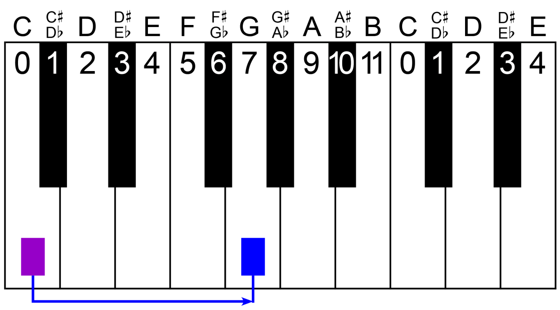
A P5 will always be the distance of 7 *half-steps*. If p is our starting point, then p+7 will be a P5. The mathematical representation for that will be:

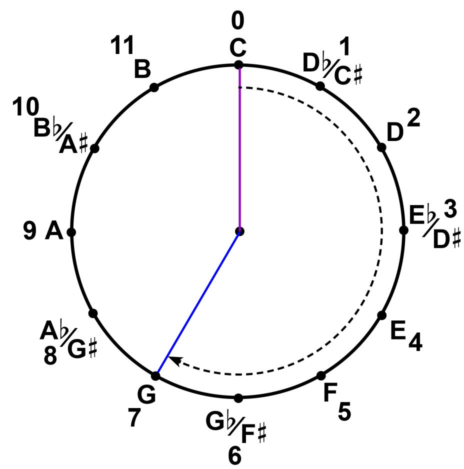
P5 = {p,(p+7)} (mod 12)

Let's compare a P5 from C and a P5 from E♭.

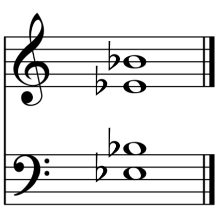
If p = 0   P5 = {0,(0+7)} (mod 12)   P5 = {0,7} (mod 12)

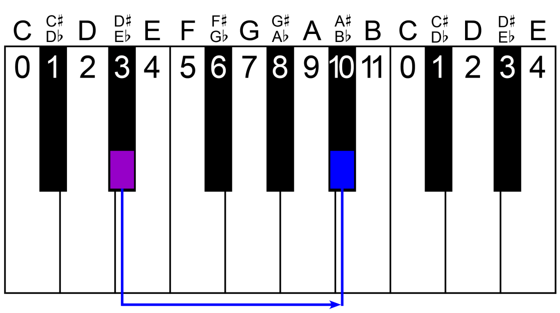


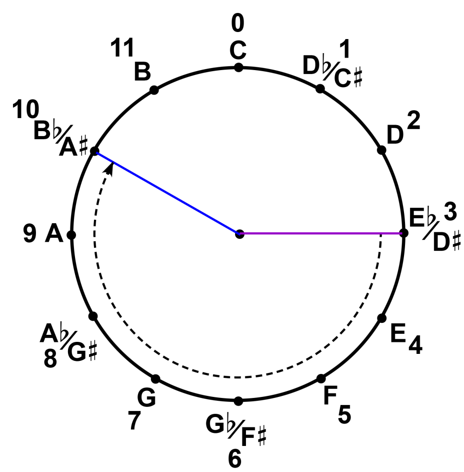




If p = 3   P5 = {3,(3+7)} (mod 12)   P5 = {3,10} (mod 12)







♫ In many respects, figuring *interval* distances from a given *pitch* is like running a mile. It doesn't matter where the starting line is, the distance to the finish line is still a mile away. Whatever the starting point is for a given *interval*, the other *pitch* will always be exactly the same number of *half-steps* away.

Just for quick reference, let's include a chart with the *interval* designations and the formula for each.

p = any *pitch* from the Chromatic Pitch Set

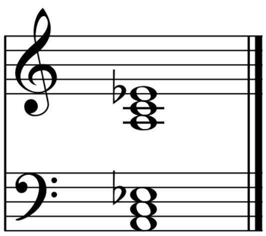
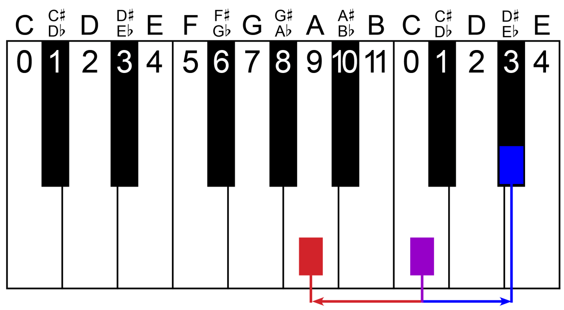
Perfect Unison	PU = {p,(p+0)} (mod 12)
Minor Second	m2 = {p,(p+1)} (mod 12)
Major Second	M2 = {p,(p+2)} (mod 12)
Minor Third	m3 = {p,(p+3)} (mod 12)
Major Third	M3 = {p,(p+4)} (mod 12)
Perfect Fourth	P4 = {p,(p+5)} (mod 12)
Augmented Fourth Diminished Fifth	A4/d5 = {p,(p+6)} (mod 12)
Perfect Fifth	P5 = {p,(p+7)} (mod 12)
Minor Sixth	m6 = {p,(p+8)} (mod 12)
Major Sixth	M6 = {p,(p+9)} (mod 12)
Minor Seventh	m7 = {p,(p+10)} (mod 12)
Major Seventh	M7 = {p,(p+11)} (mod 12)
Perfect Octave	P8 = {p,(p+12)} (mod 12)

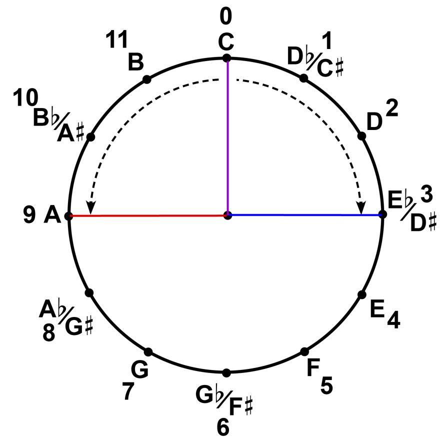
Section 4

Intervals Descending

♫ Extending the metaphor of the mile run, descending *intervals* work the same way as *ascending intervals,* they just run the same course in the opposite direction. So, our *ascending intervals* will each have a mirrored counterpart. On our CPS, the *ascending intervals* were graphed in a clockwise direction. *Descending intervals* will have the same labels and number of *half-steps*, but they will be graphed in a counter-clockwise direction.

Let's compare the same *ascending* and *descending intervals*. We'll first look at *intervals* starting on C. From C up to E♭ is a m3 (3 *half-steps*). From C down to A is also a m3 (3 *half-steps*).



The formula for this example will be:

m3 = {0, (0 ± 3)} (mod 12)

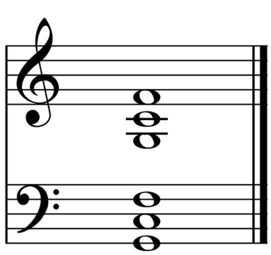
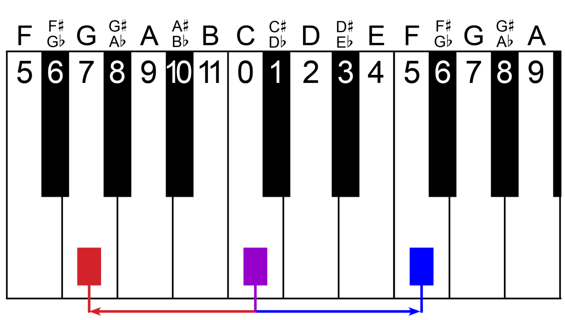
Notice the formula is the same as in the previous section, Ascending Intervals, with the exception of the ± symbol. This simply means plus or minus. So, a m3 can be either 3 *half-steps* up or 3 *half-steps* down from the given *pitch*.

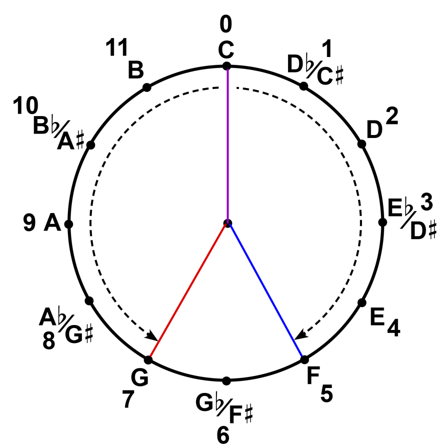
If you're looking at the actual numbers on the CPS you've noted that the *descending interval*, going counter-clockwise, counts backwards. The m3 up from C (0) is E♭ (3), and the m3 down from C (0) is A (9). So, in this instance… m3 = {0,3} or {0,9} (mod 12).

If we use the mathematical symbol for "or" the equation would look like this:

m3 = {0,3} ∨ {0,9} (mod 12)

Let's look at one more example starting from C, a P4.



The first thing you should observe is that the keyboard has been shifted over just a bit to accommodate all three of the *pitches* we're using. C (0) is in the middle of the graphic.

Here’s how this will look mathematically:

$$P4 = \{0, (0 \pm 5)\} \pmod{12}$$

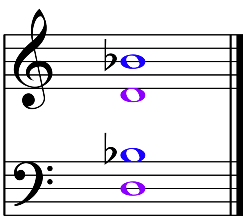
$$P4 = \{0,5\} \vee \{0,7\} \pmod{12}$$

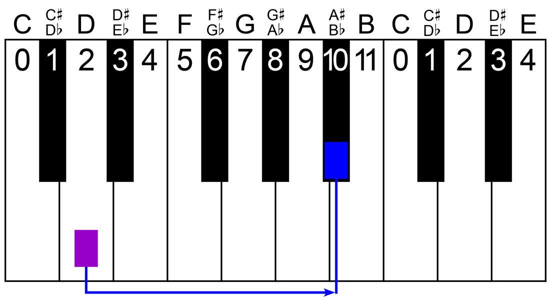
To designate whether a single *interval* should be figured as *ascending* (up) or *descending* (down) we can add one symbol to clarify the equation. To signify an *ascending interval* we will simply add the ↑ symbol to the *interval* label, such as: P4↑

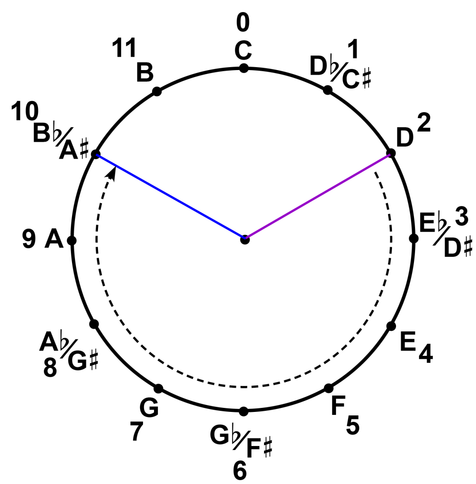
We’ll use the same procedure to indicate a *descending interval*, using the ↓ symbol, such as: P4↓

If we want to mathematically indicate an *ascending* m6 from the *pitch* D we would write:

$$m6\uparrow = \{2,10\} \pmod{12}$$

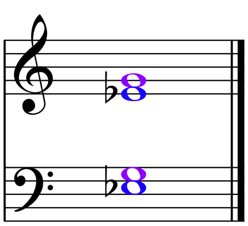


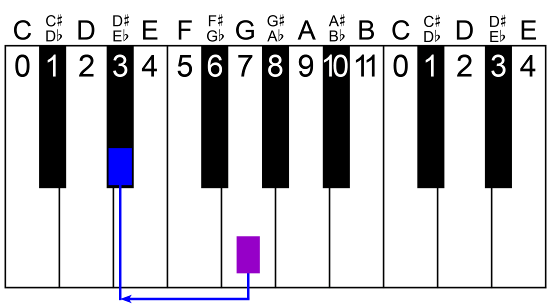


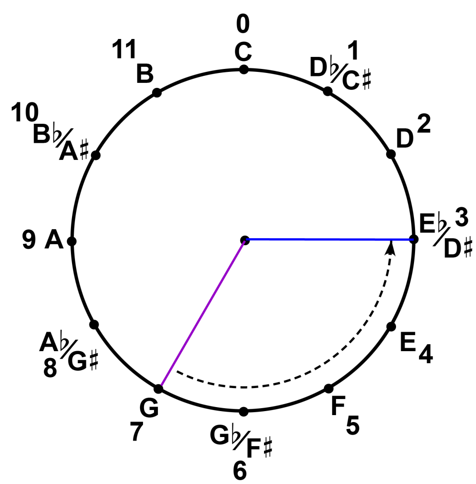


A *descending* M3 from G would be:

$$M3\downarrow = \{7,3\} \pmod{12}$$







Even though it’s a bit redundant, let’s go ahead and include another *interval* chart with the updated formula, using the ± symbol. (see the next page)

Interval Equation Chart

p = any *pitch* from the Chromatic Pitch Set

Perfect Unison	PU = {p,(p ± 0)} (mod 12)
Minor Second	m2 = {p,(p ± 1)} (mod 12)
Major Second	M2 = {p,(p ± 2)} (mod 12)
Minor Third	m3 = {p,(p ± 3)} (mod 12)
Major Third	M3 = {p,(p ± 4)} (mod 12)
Perfect Fourth	P4 = {p,(p ± 5)} (mod 12)
Augmented Fourth Diminished Fifth	A4/d5 = {p,(p ± 6)} (mod 12)
Perfect Fifth	P5 = {p,(p ± 7)} (mod 12)
Minor Sixth	m6 = {p,(p ± 8)} (mod 12)
Major Sixth	M6 = {p,(p ± 9)} (mod 12)
Minor Seventh	m7 = {p,(p ± 10)} (mod 12)
Major Seventh	M7 = {p,(p ± 11)} (mod 12)
Perfect Octave	P8 = {p,(p ± 12)} (mod 12)

Section 5

Intervals
Inversions

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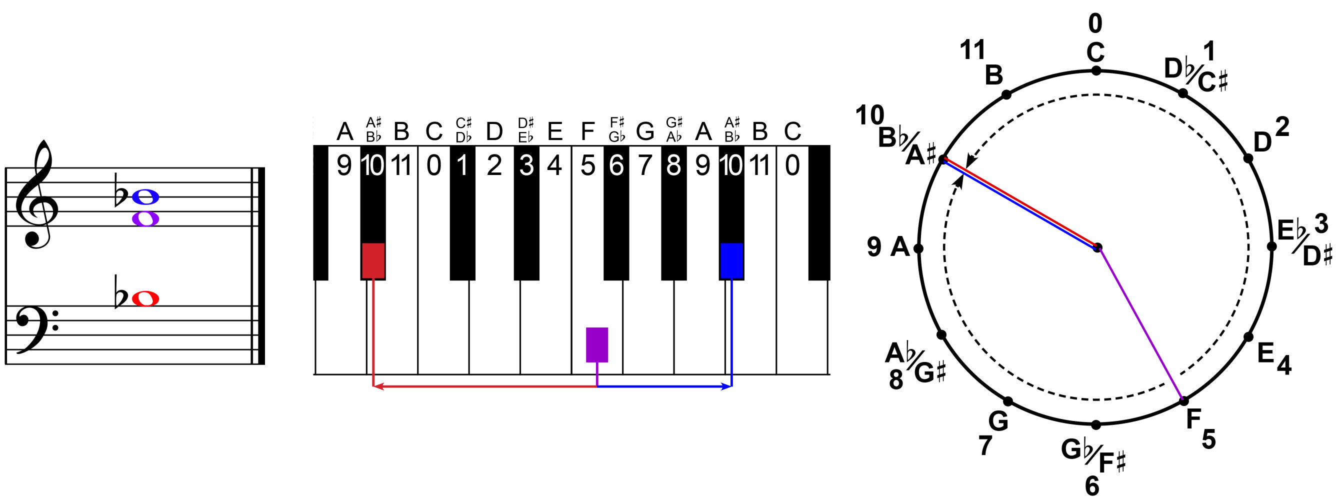
♫ Each *interval* also has a complimentary *interval* known as its *inversion*. The *inversion* is the *interval* that, when combined with the original *interval*, makes a complete *octave*. The *inversion* is the remainder of the *octave*. In numerical terms, the *inversion* will be the difference between the original *interval* and 12, remembering that 12 is congruent with 0 in modulo 12.

$$12 \equiv 0 \pmod{12}$$

♫ The *inversion* is complimentary to the original *interval* in terms of direction. If the original is an *ascending interval* the *inversion* will be *descending*. The opposite of that will be true as well. If the original is *descending*, the *inversion* will be *ascending*. On our CPS, the *inversion* will travel from the given *pitch*, in the opposite direction from the original *interval*, until it reaches the second *pitch* of the original *interval*. This concept, in my opinion, is much easier to grasp with graphic representations.

If the given *interval* is a P4↑ the *inversion* will be a P5↓ . Let's see how that looks with F as the given *pitch*. (Note : The given *pitch* is purple, the original *interval* is blue, and the *inversion* is red.)



If the original *interval* is a P4, the number of *half-steps* will be 5. Since the *inversion* will be the difference between the original *interval* and the *octave*, we will subtract 5 from 12 and get 7. The remainder (7) will be the *inversion*.

If we let the letter O represent the original *interval* and I represent the *inversion*, we could say:

If O = 5 then I = (0 - 5)
O = 5 → I = 7

In mathematical terms, the → represents the "if, then" relationship.

Just for a moment, let's think about pizza. If we have a large pizza, cut in to 12 equal slices, and someone takes 3, how much is left for the rest of us? Of course, if someone takes 3 of the 12 slices there will be 9 slices left. The same is true with *intervals* and their *inversions*. When an *interval* takes its portion of the *octave*, whatever is left is the *inversion*.

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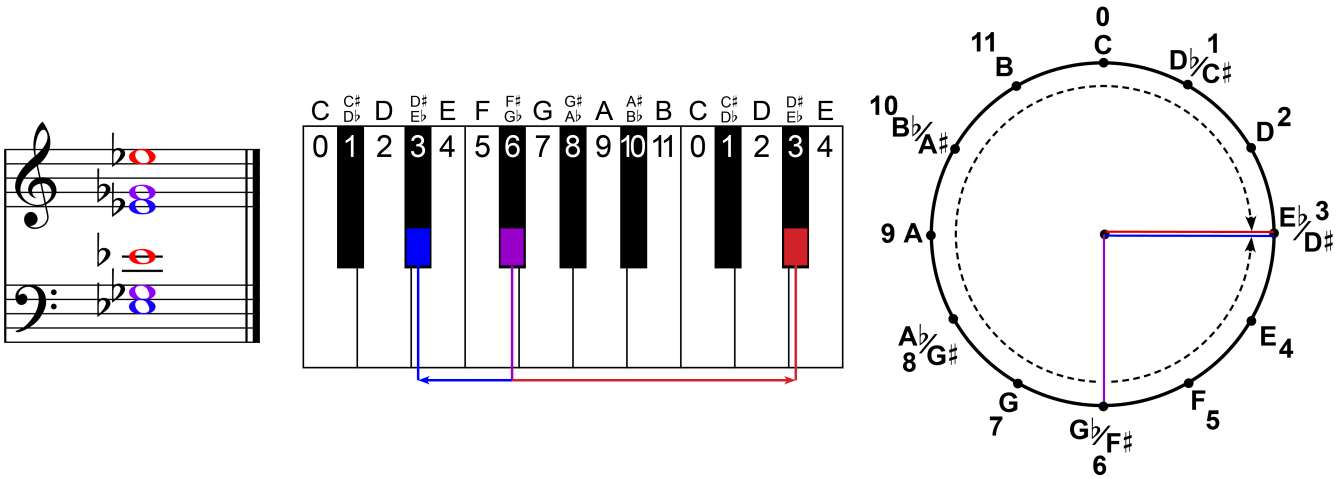
Let's look at one more graphic example of an *interval* and its *inversion*. This time we'll start with a *descending interval* as the original. We'll use a m3↓. Since the m3 is 3 *half-steps*, the *inversion* will be 9 *half-steps* or M6↑ (12 - 3 = 9).

O = 3 → I = 9

(Note: As before, the given *pitch* is purple, the original *interval* is blue, and the *inversion* is red.)



Here are a few tips to keep in mind when figuring *inversions*:

- ♫ If the original *interval* is a *perfect* (P) *interval*, the *inversion* will also be *perfect* (P). Only *octaves*, *4ths* and *5ths* can be considered *perfect*. (They can also be *diminished* or *augmented*, but we'll talk about these a bit later.)
  - P4↑ → P5↓
  - P5↑ → P4↓
- ♫ If the original *interval* is a *major* (M) *interval*, the *inversion* will always be *minor* (m), and vice-versa. *2nds*, *3rds*, *6ths*, and *7ths* can be *minor* or *major*. (These can also be *diminished* or *augmented*… later.)
  - M3↑ → m6↓
  - m2↑ → M7↓
- ♫ *Perfect unisons* and *octaves* have no real *inversions*. Each consists of the same two *pitches*, either in the same place or displaced by an *octave*. Some folks will claim that a *unison* will invert to a full *octave*, and vice-versa. Instead of discussing those issues here, we'll go ahead and *invert* those as some would suggest (see table below).

Just for reference, let's add a table here with the basic *intervals* and their *inversions*.

PU	M2	M3	P4	A4*	P5	M6	M7	P8
P8	m7	m6	P5	d5*	P4	m3	m2	PU

(See Interval/Inversion Equations Chart)

* The *augmented 4th* (A4) and *diminished 5th* (d5) will be discussed in Section 7.

Section 6

Intervals
Diatonic

If you've thought about our discussion of the *tritone* from Section 3, you've probably wondered about *augmented* and *diminished intervals*. It's about time to consider those and how they work within our system, but we first need to take a brief look at how *intervals* work within a *diatonic* setting.

For this section, we will be bringing our musical alphabet and modulo 7 back into the discussion. Let's put the CPS aside for a bit and just use the letters of our musical alphabet as reference points, but let's arrange them from C to C (like the CPS). This will form a *diatonic scale*. Do you remember our discussion of a *diatonic scale* from Section 1? If not, here's a reminder:

> ♫ A (*diatonic*) *scale* in music is a sequence of *pitches* that encompasses all seven of the letters of the musical "alphabet." A (*diatonic*) *scale* can begin on any of the seven *pitches* (letters) and progress through the remaining *pitches*.

For reference in our current discussion, here's our *diatonic scale* based on C:

C	D	E	F	G	A	B	C
$\hat{1}$	$\hat{2}$	$\hat{3}$	$\hat{4}$	$\hat{5}$	$\hat{6}$	$\hat{7}$	$\hat{1}$

If we have an *interval* that includes two adjacent letters of the musical alphabet, whether *ascending* or *descending*, it will be considered a *2nd*. Please note that there is no quality designation assigned. So, C to D, F to G, E to D, and C to B are all considered *2nds* (quality excluded).

Using a mathematical equation, with p as our given *note*, that relationship would look like this (Remember that we are looking at *scale degrees* in mod 7 and not *half-steps* as in mod 12):

$$2\text{nd} = \{p, (p\pm 1)\} \pmod 7$$

If $p = \hat{4}$, then the equation would be:

$$\{\hat{4},\uparrow\hat{5}\} \vee \{\hat{4},\downarrow\hat{3}\} = 2\text{nd} \pmod 7$$

$$p = \hat{4} \rightarrow \{\hat{4},\uparrow\hat{5}\} \vee \{\hat{4},\downarrow\hat{3}\} = 2\text{nd} \pmod 7$$

Remember that → is the symbol that represents an "if, then" relationship, and ∨ represents "or." Putting that last equation in a verbal form, it would read:

If p equals the *fourth scale degree*, then up from *scale degree* four to five, or down from four to three, would equal the *diatonic interval* of a *second*.

As mentioned above, we can see that adjacent *scale degrees* form *2nds*. So, an *interval* that has one *scale degree* in between will be a *3rd*. …two *scales degrees* in between will be a *4th*. …three *scale degrees* in between will be a *5th*, and so on.

A quick and easy point of reference would be to base each *interval* on $\hat{1}$. In this instance, from $\hat{1}$ up to $\hat{2}$ would be a *2nd*. …from $\hat{1}$ up to $\hat{3}$ would be a *third*. …from $\hat{1}$ up to $\hat{4}$ would be a *4th*, and so on. That's certainly easy to remember, but not all *intervals* have $\hat{1}$ as the given *note*, and not all *intervals* will be *ascending*.

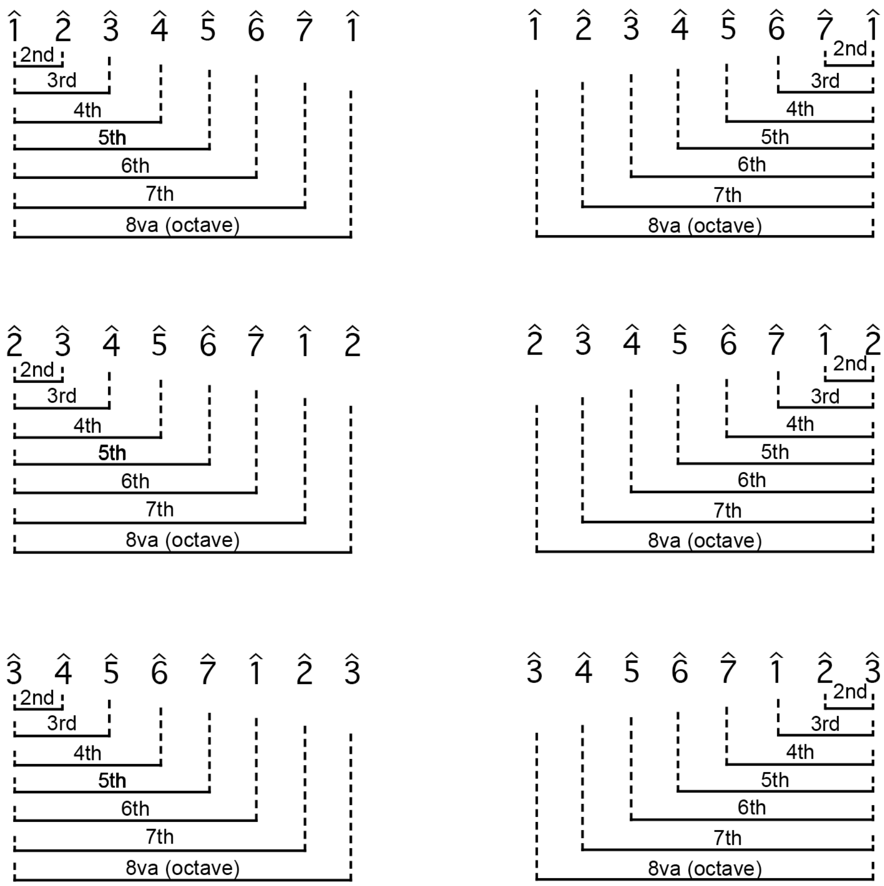
Putting this concept into a visual form should make it easier to grasp. Let's consider the following graphic representation (next page).

Diatonic Intervals

Ascending from $\hat{1}$ / Descending from $\hat{1}$

Ascending from $\hat{2}$ / Descending from $\hat{2}$

Ascending from $\hat{3}$ / Descending from $\hat{3}$



…and so on.

Using this chart for reference, we can see that:

- from $\hat{2}$ up to $\hat{4}$ is a *3rd*
- from $\hat{2}$ up to $\hat{6}$ is a *5th*
- from $\hat{2}$ down to $\hat{5}$ is a 5*th*
- from $\hat{3}$ up to $\hat{1}(\hat{8})$ is a *6th*
- from $\hat{3}$ down to $\hat{7}$ is a 4*th*
- from $\hat{3}$ down to $\hat{4}$ is a 7*th*

I'm sure we would agree, all of the *interval* relationships are easier to recognize if they have $\hat{1}$ as their given *pitch*. Let's look at these in mathematical terms. (Remember that we are looking at *scale degrees* in mod 7 and not *half-steps* as in mod 12.)

$\{\hat{1},\uparrow\hat{2}\} \vee \{\hat{1},\downarrow\hat{7}\}$ = 2nd (mod 7)
$\{\hat{1},\uparrow\hat{3}\} \vee \{\hat{1},\downarrow\hat{6}\}$ = 3rd (mod 7)
$\{\hat{1},\uparrow\hat{4}\} \vee \{\hat{1},\downarrow\hat{5}\}$ = 4th (mod 7)
etc.

If we use the letter names from our *diatonic* C scale (two pages back) and verbalize the equations (immediately above) they would read:

C up to D or C down to B are *2nds*
C up to E or C down to A are *3rds*
C up to F or C down to G are *4ths*
etc.

Of course, the relationships will be the same no matter what the particular *diatonic scale* or given *pitch* might be.

Let's add a *diatonic inversion* table here for reference.

Diatonic Interval Equations

Ascending Interval	**Descending Interval**
Unison = {p, (p+0)} (mod 7)	Unison = {p, (p-0)} (mod 7)
↑2nd = {p, (p+1)} (mod 7)	↓2nd = {p, (p-1)} (mod 7)
↑3rd = {p, (p+2)} (mod 7)	↓3rd = {p, (p-2)} (mod 7)
↑4th = {p, (p+3)} (mod 7)	↓4th = {p, (p-3)} (mod 7)
↑5th = {p, (p+4)} (mod 7)	↓5th = {p, (p-4)} (mod 7)
↑6th = {p, (p+5)} (mod 7)	↓6th = {p, (p-5)} (mod 7)
↑7th = {p, (p+6)} (mod 7)	↓7th = {p, (p-6)} (mod 7)
↑8th = {p, (p+7)} (mod 7)	↓8th = {p, (p-7)} (mod 7)

Now that we've taken a stroll through *intervals* in a *diatonic* setting, let's move on to *augmented* and *diminished intervals*.

Section 7

Intervals
Augmented and Diminished

Now that we understand *intervals* in a *diatonic* setting, let's add a bit of a *chromatic* twist to them. For this we'll use our *diatonic* relationships as a starting point but go to our CPS as a resource.

If you remember our discussion of the *tritone*, and how two *intervals* can have the same number of *half-steps* but two different labels, here's our chance to sort that out.

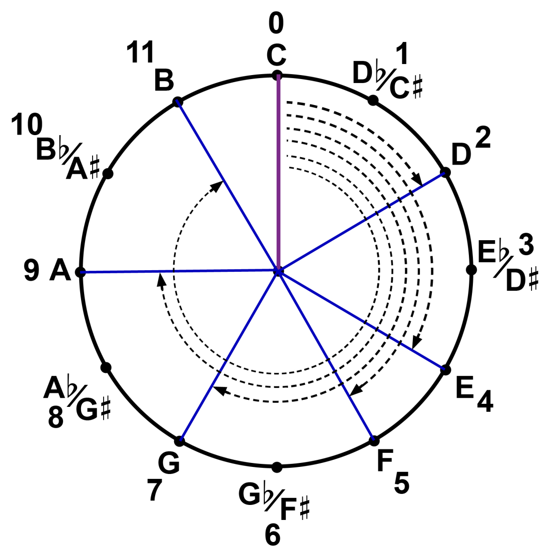
First, let's remind ourselves of the *intervals* we've covered so far. Here's our *half-step* chart:

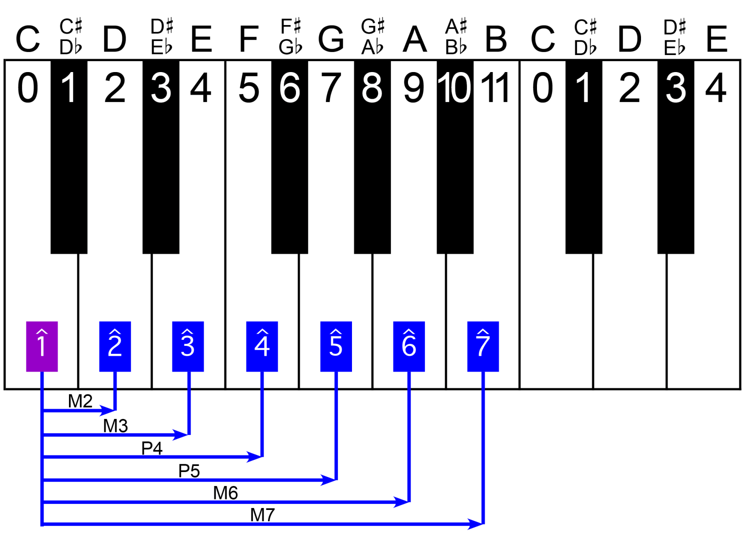
LABEL	PU	m2	M2	m3	M3	P4	A4 d5	P5	m6	M6	m7	M7	P8
# of half-steps	0	1	2	3	4	5	6	7	8	9	10	11	12

Also, keep the CPS close by to refer to as we go. In fact, let's just jump in and use it as our starting point. First, we'll look at just the *pitches* and *intervals* that have no *accidentals* (*sharps* or *flats*) indicated, then we'll come back and sort out the *enharmonic* spellings.

Using the *diatonic* letter names, from C clockwise around to B, we have what is referred to as a C *major scale* (we'll cover *scales* in more depth later). This *scale* gives us the basis for quite a bit of what we'll be covering throughout our study. For now, let's just consider the *intervals* that are included. The *ascending intervals* from C are:

- C up to D is a M2 — $(\hat{1}, \uparrow\hat{2})$ = M2
- C up to E is a M3 — $(\hat{1}, \uparrow\hat{3})$ = M3
- C up to F is a P4 — $(\hat{1}, \uparrow\hat{4})$ = P4
- C up to G is a P5 — $(\hat{1}, \uparrow\hat{5})$ = P5
- C up to A is a M6 — $(\hat{1}, \uparrow\hat{6})$ = M6
- C up to B is a M7 — $(\hat{1}, \uparrow\hat{7})$ = M7



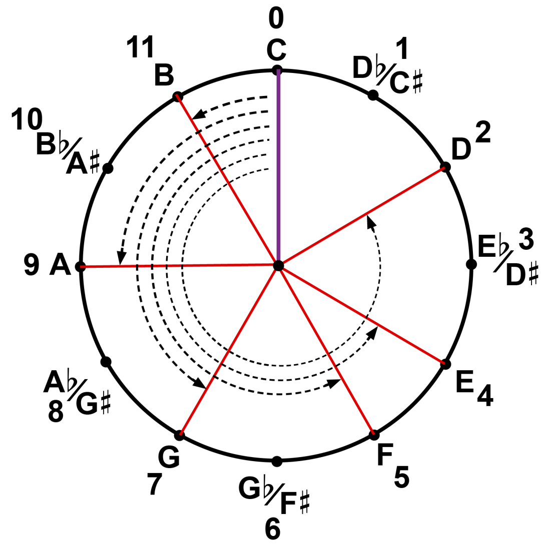


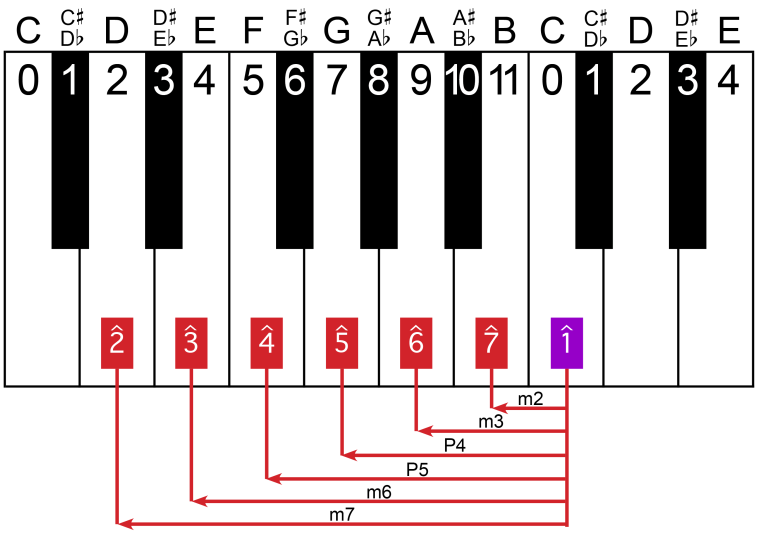
All of these *pitches* and *intervals* are in a *natural* state (no *accidentals*). That is probably the main reason we use this *scale* as an ongoing point of reference.

Let's not forget our *descending intervals*, using the C *major scale* again.

The *descending intervals* from C are:

- C down to B is a m2 $(\hat{1},\ \downarrow\hat{7})$ = m2
- C down to A is a m3 $(\hat{1},\ \downarrow\hat{6})$ = m3
- C down to G is a P4 $(\hat{1},\ \downarrow\hat{5})$ = P4
- C down to F is a P5 $(\hat{1},\ \downarrow\hat{4})$ = P5
- C down to E is a m6 $(\hat{1},\ \downarrow\hat{3})$ = m6
- C down to D is a m7 $(\hat{1},\ \downarrow\hat{2})$ = m7

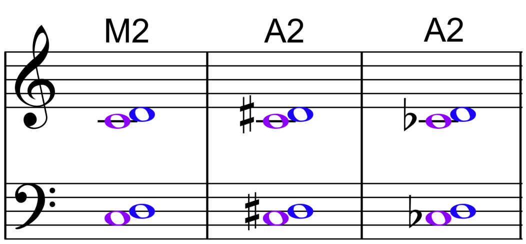




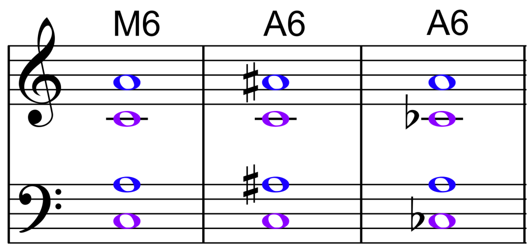
Taking these *intervals* and altering them *chromatically*, by adding *accidentals* (*sharps* or *flats*), will require us to give them new labels. Let's look at how the alterations will change the labels.

♬ If the *interval* is *major* (M), increasing the number of *half-steps* by 1, without changing the letter names, will make the *interval augmented* (A). Here are two examples:

C up to D is a M2; C up to D♯ is an A2; C♭ up to D is also an A2

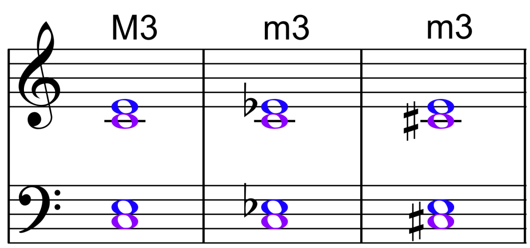


C up to A is a M6; C up to A♯ is an A6; C♭ up to A is also an A6

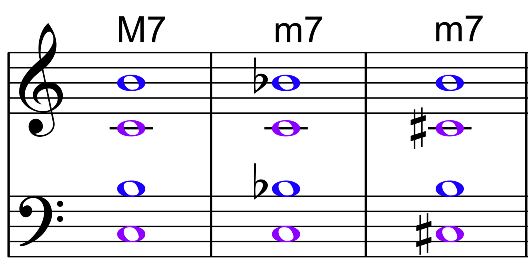


♬ If the *interval* is *major* (M), decreasing the number of *half-steps* by 1, without changing the letter names, will make the *interval minor* (m). Here are two examples:

C up to E is a M3; C up to E♭ is an m3; C♯ up to E is also a m3



C up to B is a M7; C up to B♭ is a m7; C♯ up to B is also a m7



If you're following this closely you might be thinking… "C to D♯ are the same *notes* as C to E♭. They certainly look the same on the CPS and a keyboard. Why do they have different labels?"

The reason lies in the *diatonic* letter names. C up to D is *diatonically* a *2nd*, no matter what *accidentals* are added. C up to E is *diatonically* a *3rd*, no matter what *accidentals* are added. An A2 and a m3 may look and sound the same on the CPS or a keyboard, but they will have different *functions* within a piece of music. We'll deal with *function* later on in our study.

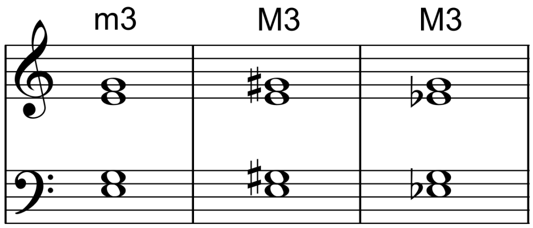
From our examples above, the same will be true of an A6 (C up to A♯) and a m7 (C up to B♭). They will look and sound the same on the CPS and a keyboard, but they will have different musical *functions*.

One of the reasons we add graphics is to give you a visual point of reference. If you just glance at each set of examples on the *staff* above you can easily see the *intervals* in each set look the same distance apart. Of course, on the *staff* they actually are. The *accidentals* are what change the labels.

Let's move on. Here's the rest of the combinations you'll need to know for now.

♫ If the *interval* is *minor* (m), increasing the number of *half-steps* by 1, without changing the letter names, will make the *interval major* (M).

E up to G is a m3; E up to G♯ is a M3; E♭ up to G is also a M3

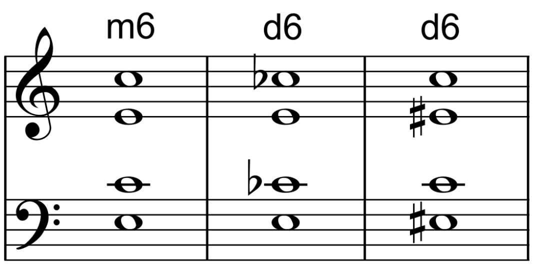


Please note: The *intervals* are the same if you read them from the top *note* down.

G down to E is a m3; G♯ down to E is a M3; G down to E♭ is also a M3

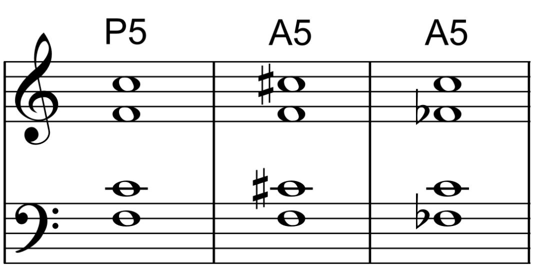
♫ If the *interval* is *minor* (m), decreasing the number of *half-steps* by 1, without changing the letter names, will make the *interval diminished* (d).

E up to C is a m6; E up to C♭ is a d6; E♯ up to C is also a d6



♫ If the *interval* is *perfect* (P), increasing the number of *half-steps* by 1, without changing the letter names, will make the *interval augmented* (A).

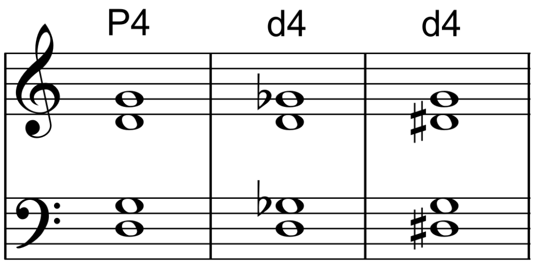
C down to F is a P5; C♯ down to F is an A5; C down to F♭ is also an A5



♫ If the *interval* is *perfect* (P), decreasing the number of *half-steps* by 1, without changing the letter names, will make the *interval diminished* (d).

G down to D is a P4; G♭ down to D is a d4; G down to D♯ is also a d4

(next page)



Below is a table to help you visualize the *interval* changes noted above. It can be read from the middle column (Original Interval) to either the left (Decreased) or right (Increased). Remember, the letter names of the *pitches* will remain the same (see above).

It can also be read left to right, starting with the left-hand column, adding 1 *half-step* for each column to the right.

Right to left is an option as well, subtracting 1 *half-step* for each column to the left.

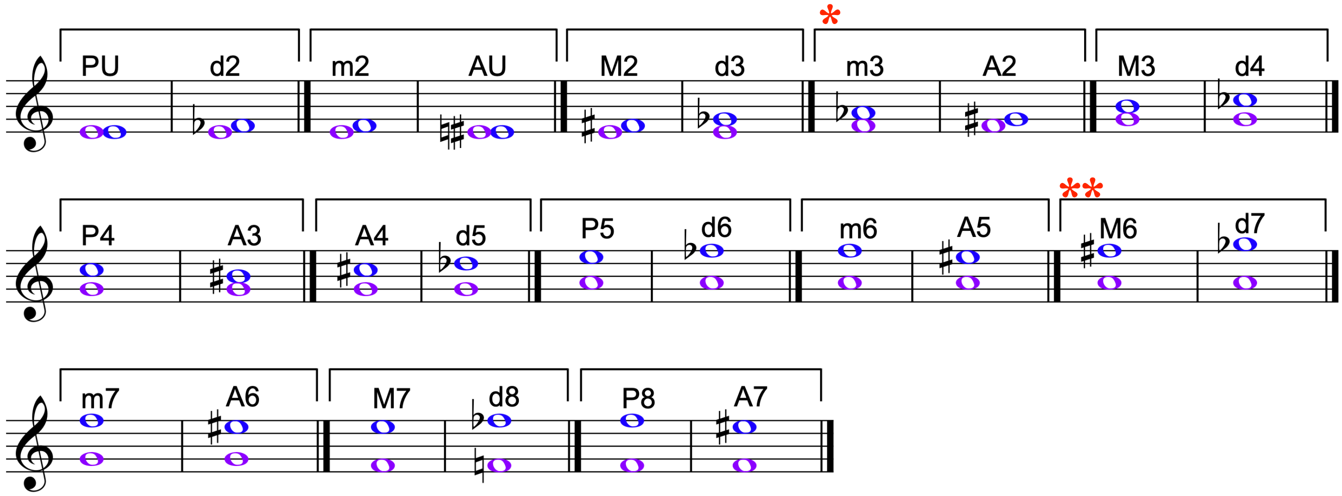
Decreased by 1 half-step	**Original Interval**	**Increased by 1 half-step**
d (diminished)	P (perfect)	A (augmented)
m (minor)	M (major)	A (augmented)
d (diminished)	m (minor)	M (major)
doubly diminished (very rare)	d (diminished)	P (perfect) or m (minor) depending on the original interval
P (perfect) or M (major) depending on the original interval	A (augmented)	doubly augmented (very rare)

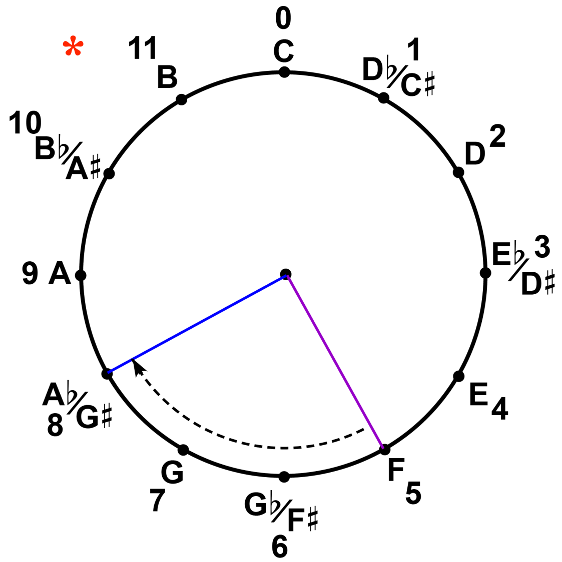
So, let's flip the coin and look at these from the other side. Above we considered how the *accidentals* changed the quality (sound) of a given *diatonic interval*. Now let's look at *enharmonic intervals* that change the *diatonic intervals* but not the sound or how they appear on the keyboard.

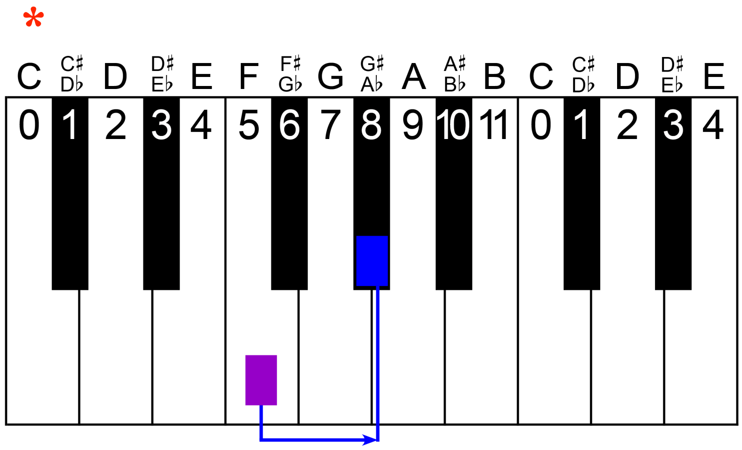
First, let's update our *half-step interval* chart to include the *enharmonic* variables.

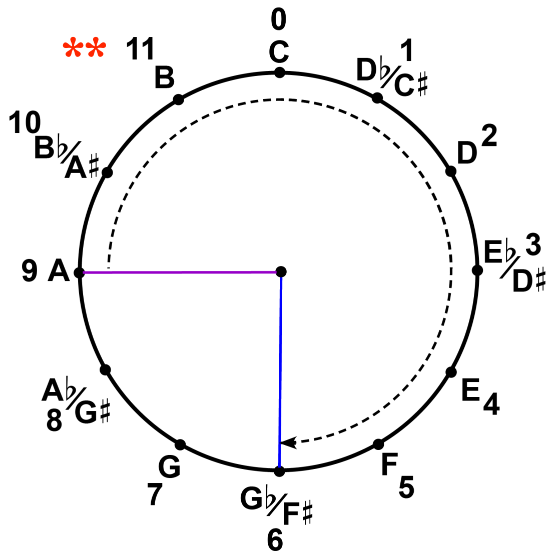
LABEL	PU d2	m2 AU	M2 d3	m3 A2	M3 d4	P4 A3	A4 d5	P5 d6	m6 A5	M6 d7	m7 A6	M7 d8	P8 A7
# of half-steps	0	1	2	3	4	5	6	7	8	9	10	11	12

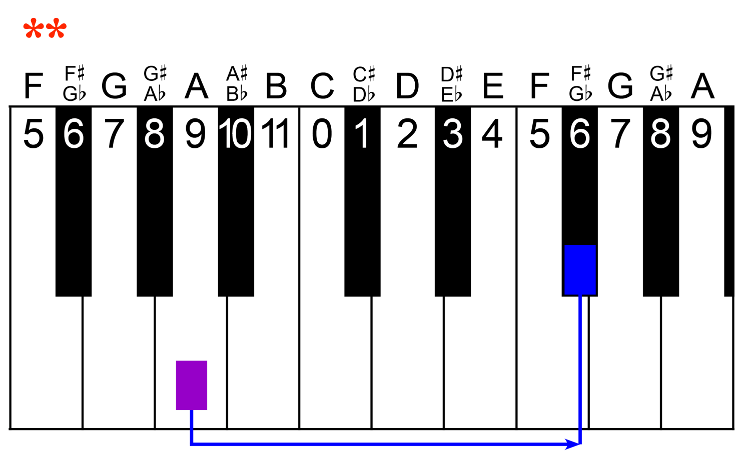
Here are some musical examples of these *enharmonic intervals*. Each pair is bracketed together as a set. Notice that each set has two different spellings, using different *diatonic pitch* names, but when they are played on a keyboard or graphed on the CPS they appear to be the same.











There's one more issue we need to wrap up. Since we have added the *augmented* and *diminished intervals* to our vocabulary, let's look at how they fit into the context of *inversions*. A couple of charts and examples will probably be sufficient to see how all of this works together.

Interval Quality	Inverts to	Interval Quality
P	↔	**P**
M	↔	**m**
A	↔	**d**

Please note that these charts are bi-directional. They can be read left to right or right to left.

Interval Distance	Inverts to	Interval Distance
U	↔	**8**
2	↔	**7**
3	↔	**6**
4	↔	**5**

Here are a few examples of what these will look like with both quality and distance:

P4 ↔ P5 M3 ↔ m6 A5 ↔ d4 m7 ↔ M2 PU ↔ P8 M6 ↔ m3 A4 ↔ d5

Section 8

Diatonic Scales
Major

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Once again, let's refresh our memories of what was noted in Section 1 about *diatonic scales*.

> ♫ A (*diatonic*) *scale* in music is a sequence of *pitches* that encompasses all seven of the letters of the musical "alphabet." A (*diatonic*) *scale* can begin on any of the seven *pitches* (letters) and progress through the remaining *pitches*.

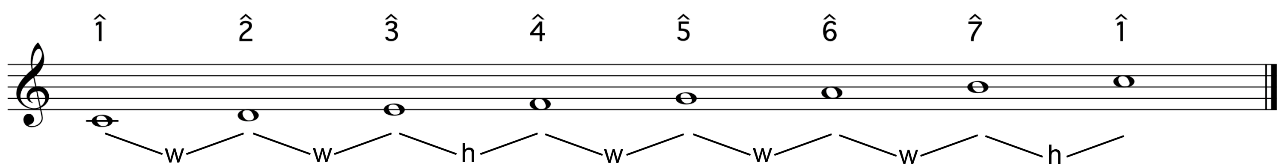
A note should be made for clarity, that each of the seven *pitches* (letter names) will only be used once in a *diatonic scale*. For example, you should not see an F and an F♯ in the same *diatonic scale*.

These sequential sets of seven *pitches* will exist within one *octave* and are organized into predetermined patterns. These patterns are generally arranged in terms of *half-steps* and *whole-steps*. As a reminder, a *half-step* is a m2 and a *whole-step* is a M2.

$\text{m2} = \{p,(p \pm 1)\} \pmod{12}$ $\quad$ $\text{M2} = \{p,(p \pm 2)\} \pmod{12}$ $\quad$ (see the equations on page 21)

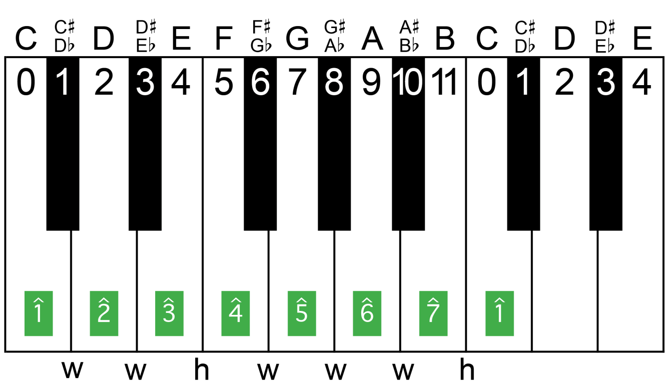
As in Section 7, we'll be using the *diatonic scale* based on C. This *scale* is typically used as a reference because it does not include any *accidentals* in its structure. This *scale*, with its arrangement of *half-steps* and *whole-steps*, is a very accessible version of what is called a *major scale*.

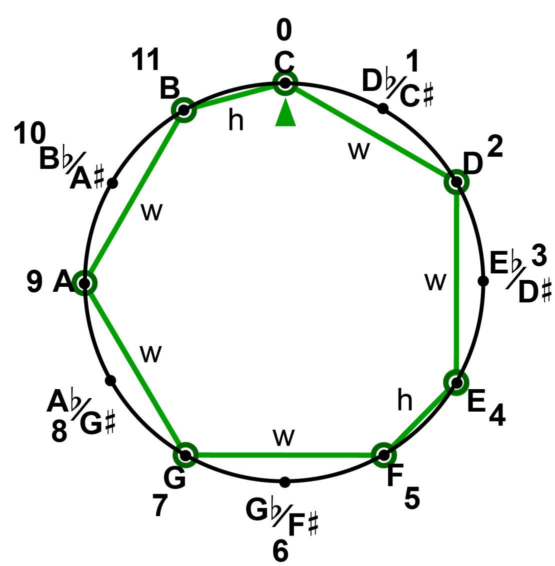
A *major scale* has a sequence of *half-steps* and *whole-steps* that gives it its unique structure and quality. Let's look at that structure in the *C major scale* below. Let's designate h for *half-step* and w for *whole-step*.



♫ This arrangement of *half-steps* and *whole-steps* will be exactly the same for any *major scale*, built on any *pitch*.

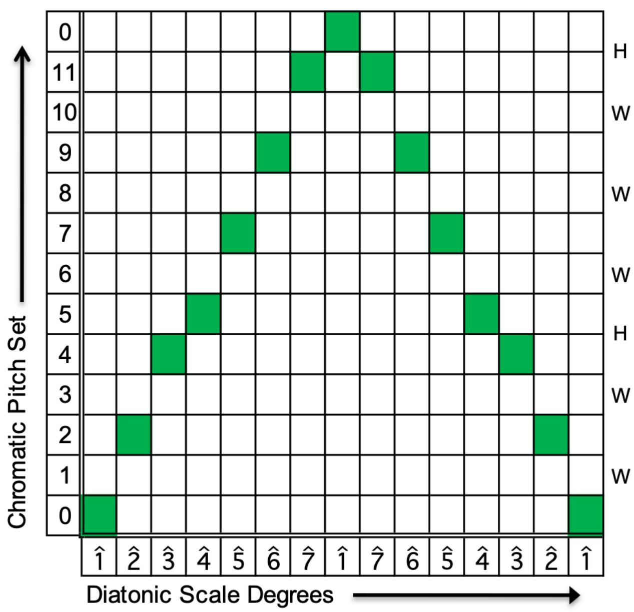
Let's look at this *scale* in some other forms. It's often easier to see the spacing of the *half-steps* and *whole-steps* in different graphic representations.





Let's add a new type of graph/chart. This one gives us a visual representation of a *scale* structure in an *ascending* and *descending* form. It also shows us how a *diatonic scale* interacts with the CPS. (next page)

C Major Scale



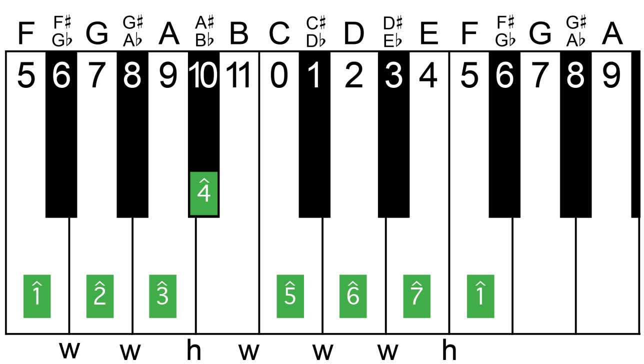
Notice the numbers of the CPS in the far left column, ascending from bottom to top. The *diatonic scale degrees* are in the bottom row, *ascending* from left to right to the middle of the graph, then descending from the middle to the far right. On the far right edge of the graph we have the arrangement of *half-steps* and *whole-steps*.

With this graph, it is easiest to see that the *scale* structure doesn't change, no matter what the starting *pitch* might be.

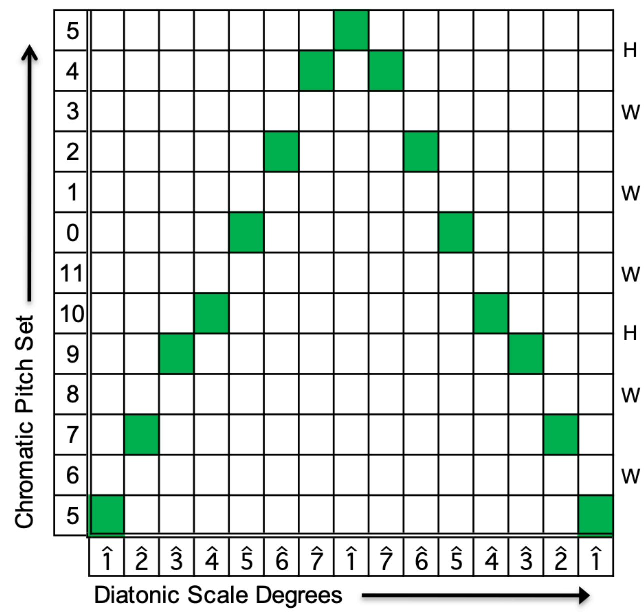
Here's one other example of our block graph that shows the *F major scale*. Notice that the only thing that has changed is the starting point on the CPS.

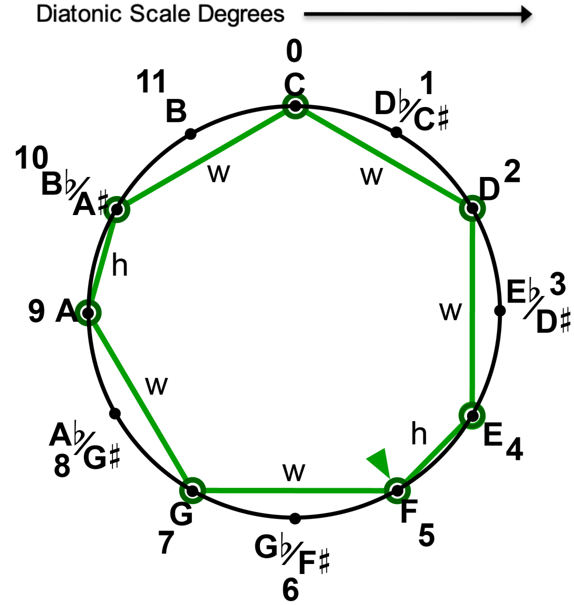
This type of graph certainly serves to reinforce the statement on the previous page, the "arrangement of *half-steps* and *whole-steps* will be exactly the same for any *major scale…*"

Even though the arrangement stays the same, the *scales* will look different on a keyboard and on the CPS. Let's look at those for the *F major scale.*



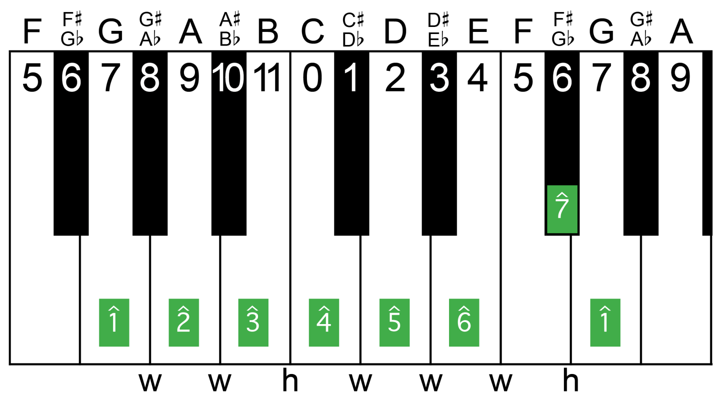
F Major Scale

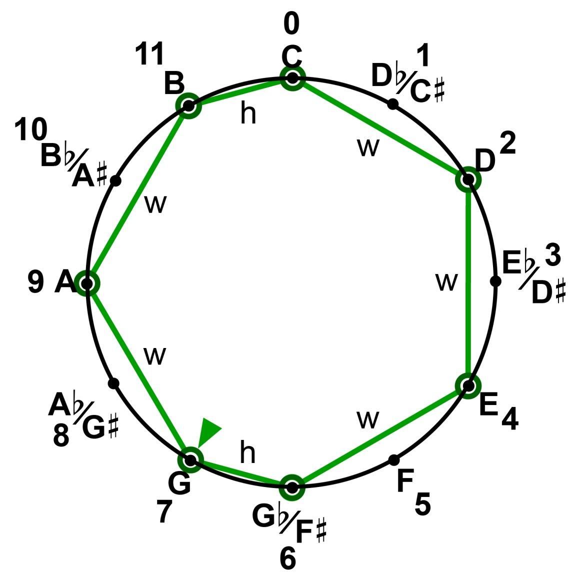




The biggest difference you probably see is the use of one of the black keys on the keyboard. You have to look a bit closer at the CPS to note the difference between *C major* and *F major*. There are two things that have changed. The first thing, of course, is the starting point of F, designated by the arrowhead. The second is the the B♭. Other than that *pitch*, the CPS as a whole looks virtually the same. If you think about it, to change from *C major* to *F major* we just need to change B to B♭.

That basic observation will hold true for *G major* as well. We will only have to change one *pitch* to shift from *C major* to *G major*. Looking at the keyboard, we can easily see that the F from *C major* will become F♯ in *G major*. The same is true with the CPS, although it's not quite as obvious.





♫ When a piece of music is based on a specific *major scale*, we say that the piece is in that *key*. If a piece is based on a *C major scale*, we say it is in the *key* of *C major*. If the *F major scale* is the basis for a piece, the *key* will be *F major*.

♫ Also, the *pitch* (or *tone*) the *scale* is based on will be referred to as the *tonic pitch* or the *tonal center*. For example, if *G major* is the *scale* and the *key*, then G will be the *tonic pitch/tonal center*. These concepts of *keys* and *tonal centers* will be discussed more completely in Section 10.

One issue that hasn't been addressed is why these scales we've been discussing are referred to as "major." At this point in our study, let's just say that the label is mostly based on the *3rd scale degree* ($\hat{3}$). In the *major scale*, the *interval* from the *tonic pitch* ($\hat{1}$) to the *3rd* ($\hat{3}$) is a *major 3rd* (M3). Also, if you remember our discussion of *diatonic intervals*, you'll recall that every *ascending interval* of a *major scale*, from the *tonic pitch* ($\hat{1}$), is either *major* (M) or *perfect* (P). There will be other justifications for the label when we reach our discussion of *diatonic chords*.

For now, let's continue and discuss the construction of *diatonic major scales* from a more mathematical perspective. Since a *major scale* is a set of *pitches* in a fixed pattern of *whole-steps* and *half-steps*, an equation might be a good option for determining which *pitches* should be included in a set (*scale*), beginning with the *tonal center*.

♫ Since any *pitch* from our CPS can be used as a *tonal center*, let's designate p as the variable for that *pitch*. Let's use the uppercase M to represent "major scale" for this equation. Since the *major scale* is simply a set of *pitches*, we'll set this equation up as a mathematical set. Here's what the *major scale* would look like as a math equation:

$$M = \{p,(p+2),(p+4),(p+5),(p+7),(p+9),(p+11)\} \pmod{12}$$

Let's break this down to see what's happening. In the equation, each *pitch* is separated by a comma. So, p will be our starting point (*tonal center*). The next *pitch* is (p+2). From our discussion of *intervals*, you'll remember that p plus 2 *half-steps* is a M2 (*whole-step*). In our *scale* pattern, the next *interval* will be another *whole-step*. So, we'll add another M2 to find the next pitch in the *sequence,* and so on.

If you only look at the numbers in the formula, you should see our *scale* pattern. Instead of trying to verbalize this whole *sequence*, let's add another graphic that should help.

M = {p,(p+2),(p+4),(p+5),(p+7),(p+9),(p+11)} (mod 12)

w w h w w w

It might also help to look back at Section 7 and the chart that shows the *intervals* of a *major scale* up from the *tonic pitch*. That is what's happening here. Each consecutive *pitch* in the *sequence* is an increasingly larger *interval* from the *tonic*. Those *intervals* would be M2, M3, P4, P5, M6, and M7.

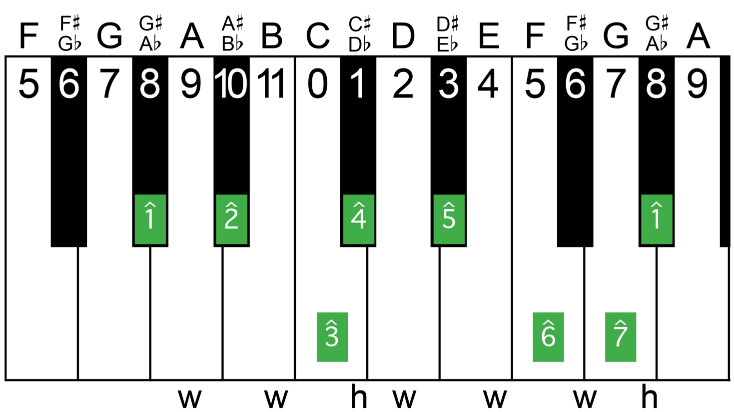
If you've noticed, there's a *half-step* missing at the end of the equation above. What's missing is the repeated *tonic pitch* at the top of the *scale*. That *pitch* would be (p+12) (an *octave*) and from (p+11) to (p+12) would be one *half-step*.

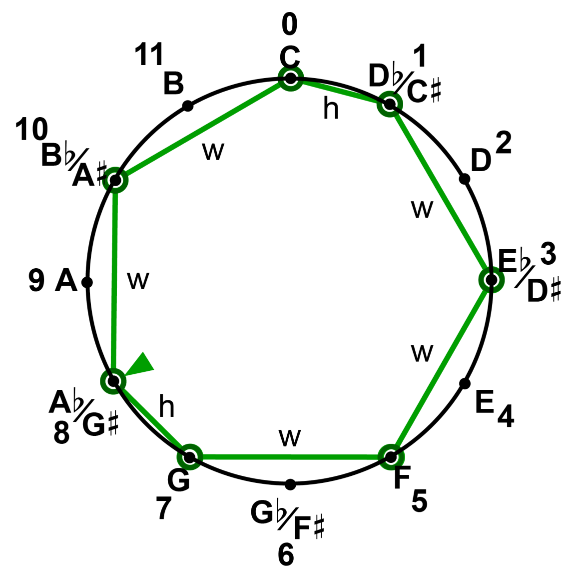
Let's put some real numbers into this equation to see how it actually works. Let's build a *major scale* on A♭. On our CPS that's 8, so p = 8.

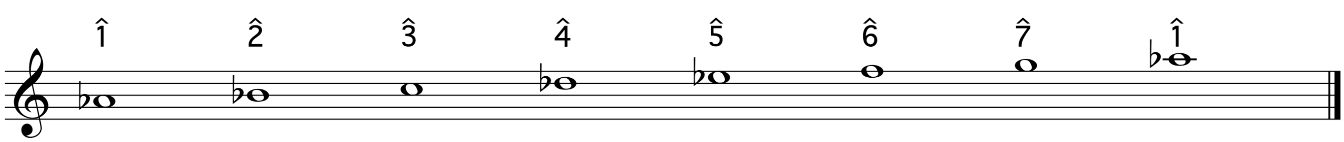
M = {8,(8+2),(8+4),(8+5),(8+7),(8+9),(8+11)} (mod 12)

Remember: Congruent numbers in Modulo 12

M = {8, 10, 0, 1 , 3, 5, 7} (mod 12)







In the next section we will focus on *minor diatonic scales*.

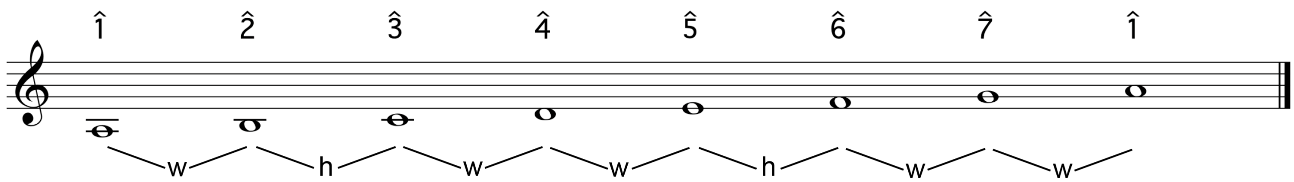
Section 9

Diatonic Scales
Minor

Now, the real fun begins. In my opinion, the *minor scales* are the most colorful. Did you notice that I said, "*scales,*" plural? That is correct. In the *tonal* tradition of the "Common Practice Period" (roughly 1650 to the early 20th century), there are three widely-accepted types of *minor scales*.

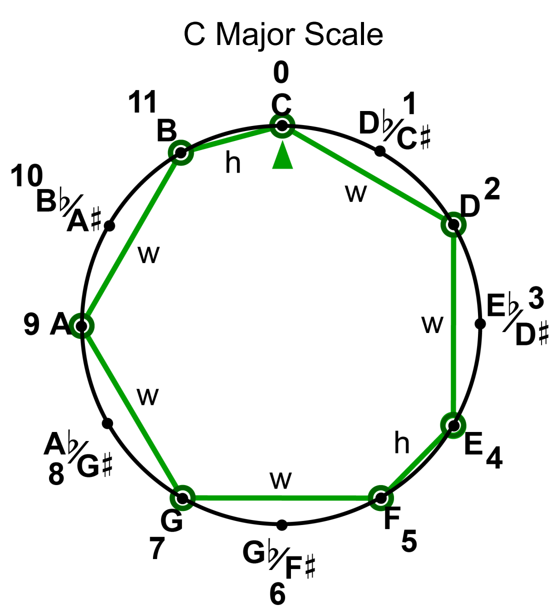
♫ Each has the same *half-step/whole-step* structure through $\hat{5}$. From the *pure* form, one *scale* alters $\hat{7}$, and the other alters both $\hat{6}$ and $\hat{7}$.

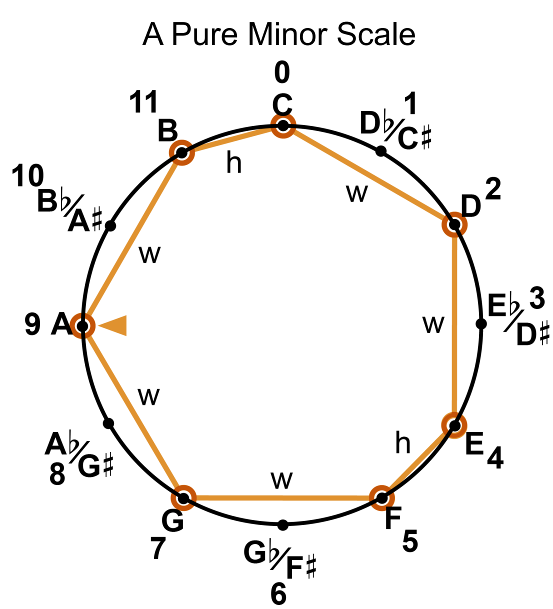
♫ The ***pure*** or ***natural minor*** is going to be our starting point. Just from the name, we should be able to tell that this is the simplest and the one that is unaltered. As with the *C major scale*, with its lack of *accidentals*, we'll start by looking at the *A pure minor scale*.



Since we discussed the reasons for calling our *major scales* "major," let's take just a quick look at a couple of reasons we call these "minor" *scales*. The first is the fact that the interval from $\hat{1}$ to $\hat{3}$ is a m3. Also, in the *pure/natural minor scale*, from $\hat{1}$ to $\hat{6}$ is a m6. Both of these reasons play significant roles in the way we will construct *harmony* and even become important in some of our advanced *harmonic* structures.

♫ With our *pure A minor scale*, one of the first things we can note is the fact that it uses all of the same *pitches* as our *C major scale*. In fact, if we compare these two scales on the CPS we'll see that they look exactly the same. The only difference is the *pitch* the *scale* is built on — the *tonal center*.

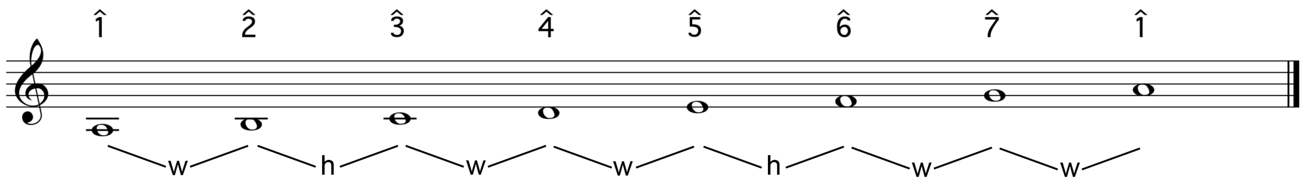




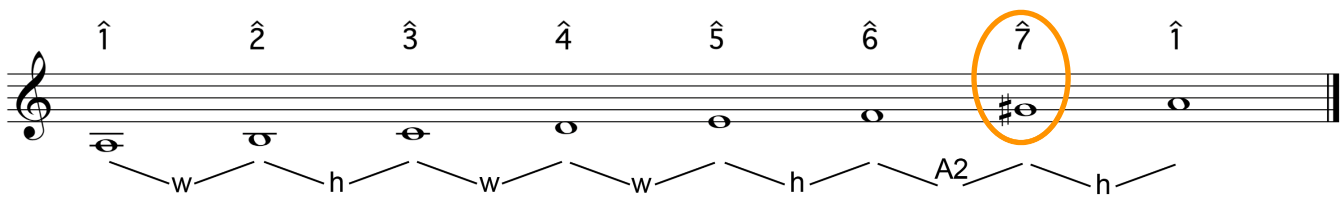
♫ The DNA of these two scales would indicate they are related. In fact, *A minor* is the *relative minor* of *C major*. *C major* is considered the *relative major* of *A minor*. This relationship will be the same for any *major scale* and the *minor scale* built on the *major scales's* $\hat{6}$. (There will more about this in future discussions.)

So, the *pure/natural minor scale* will be our basis for the other two versions. Let's first look at the musical *notation* of each, before we get to the other graphics.

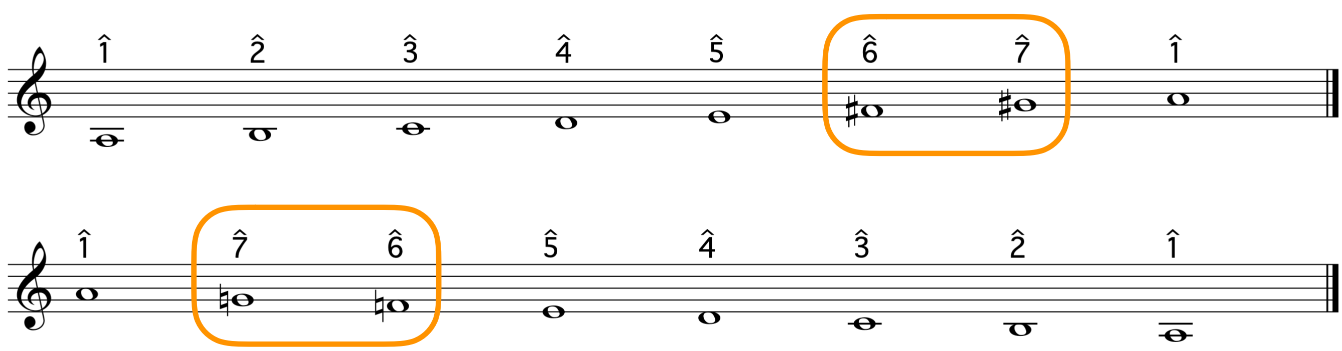
Here, again, is the *A pure minor scale*:



♫ The first alteration (second version of the *scale*) we'll look at is the ***harmonic minor.*** The name indicates that this version is altered to coincide with a *harmonic function* (more about this later). The *harmonic minor* is just like the *pure minor* with one exception, the $\hat{7}$ is raised by one *half-step*. In *pure minor*, there is a *whole-step* between $\hat{7}$ and $\hat{1}$. The *harmonic minor* changes this so the *scale* will have a *half-step* there. This raised $\hat{7}$, to our ears, leans strongly toward $\hat{1}$. It is leading us toward *resolution*. It wants to *resolve* to $\hat{1}$. When the $\hat{7}$ has this *half-step* relationship to $\hat{1}$, it is referred to as the *leading tone*.



♫ The third version of the *minor scale* is referred to as ***melodic minor***. Needless to say, composers used this option with *melodic* considerations in mind. Compared to *pure minor*, this version raises both the $\hat{6}$ and $\hat{7}$ when the line is *ascending*, but when the line is *descending*, the *scale* reverts back to the original *pitches* of the *pure minor*.



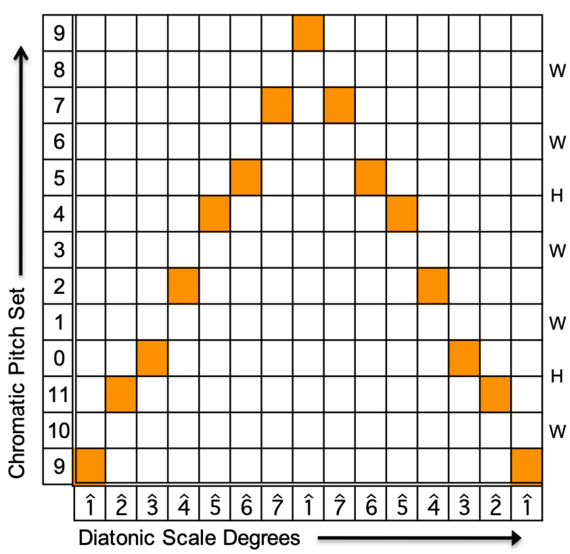
If you'll notice, the *scale degree* numbers and the actual *pitches* on the staff do not change, even though the *pitches* are altered with *accidentals*. Just looking at these on the *staff*, it is sometimes tricky to visualize what is happening to the *half/whole* patterns. Our block graphs and CPS layouts should give us a better visual representation. (next page)

Here are the block graphs for the three versions of the *A minor scale*.

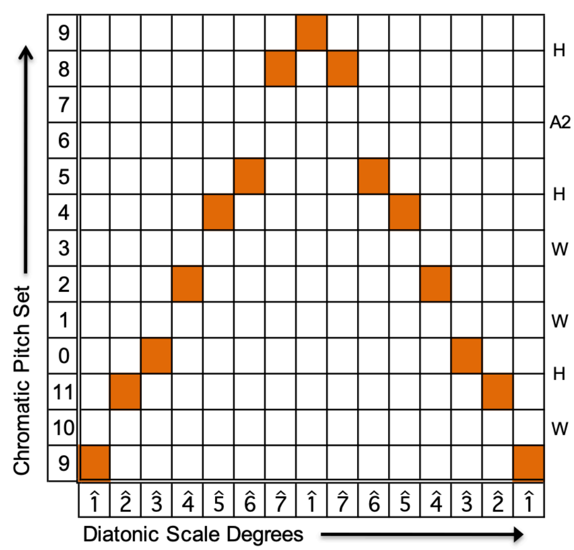
The significant change to note in *harmonic minor* is the A2 (three *half-steps*) between $\hat{6}$ and $\hat{7}$. When writing traditional (common practice period) *melodic lines*, there are certain cautions and considerations that should be noted for the A2, especially in *descending* movement (for later discussion).

In *melodic minor*, a feature that gives the *scale* its unique sound is the four consecutive *whole-steps* from $\hat{3}$ to $\hat{7}$. This is the only *scale* that incorporates that many consecutive *whole-steps*, until the latter part of the common practice period when composers began breaking away from the use of traditional *diatonic* structures.

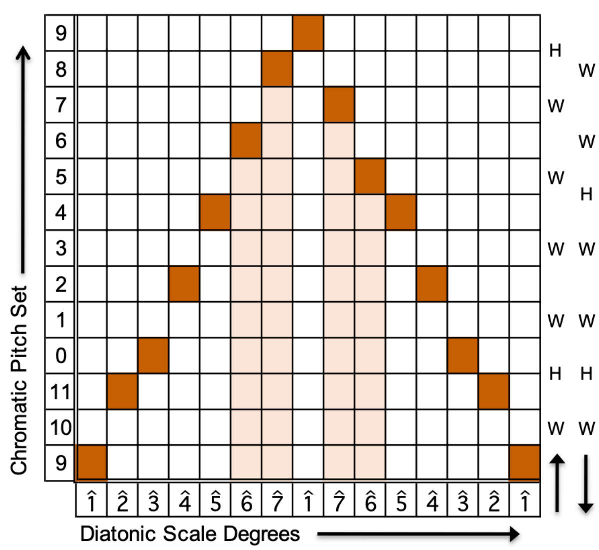
A Pure (Natural) Minor Scale



A Harmonic Minor Scale



A Melodic Minor Scale



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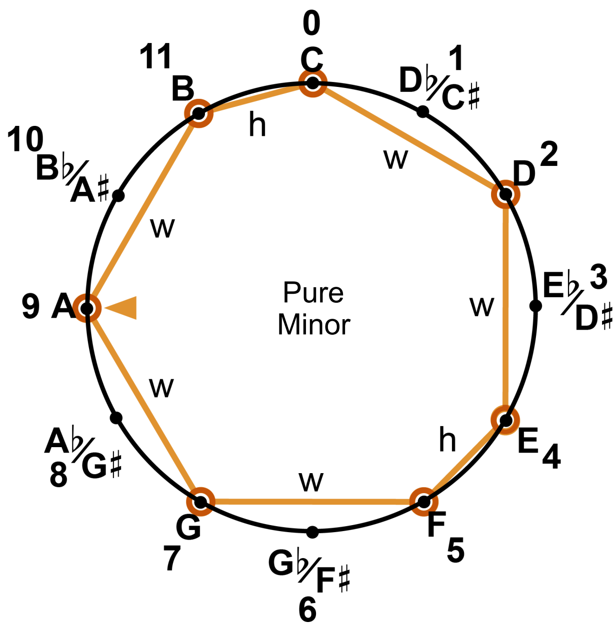
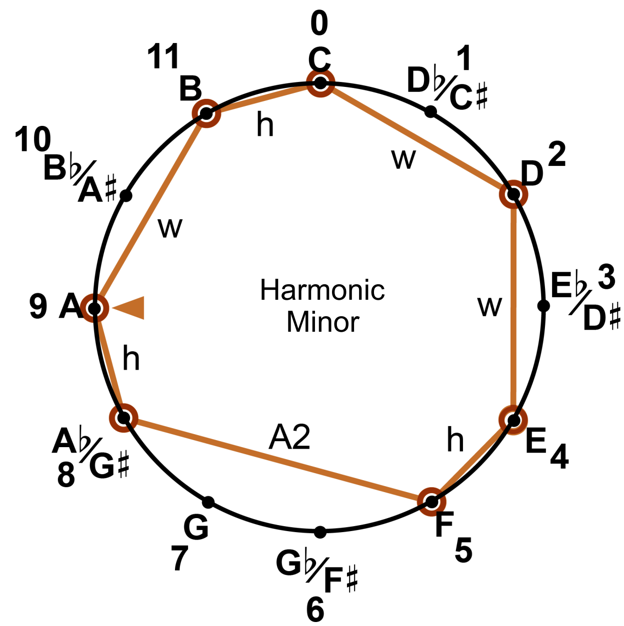
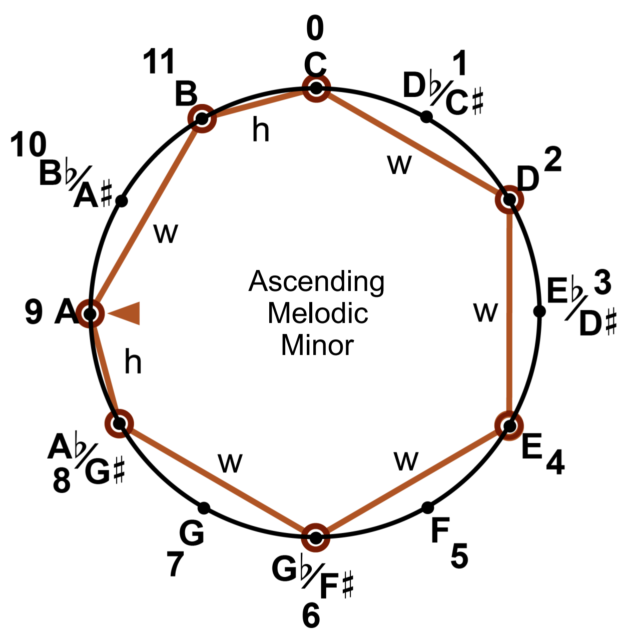
Our CPS diagrams of the three *minor* versions will also show us how the altered *scale degrees* (*accidentals*) reshape the *pure minor* form.

You'll notice that there is no CPS diagram for the *descending melodic minor*. Remember, it is exactly the same as the *pure minor*.







Before we put this section to bed, let's remind ourselves that these *scales* can be based on any *pitch* of our CPS. The patterns will be exactly the same, no matter what the *tonal center* might be. Also, for reference, the equation for a *pure minor scale* would be (using the lowercase m for *minor scale*):

$$m = \{p,(p+2),(p+3),(p+5),(p+7),(p+8),(p+10)\} \pmod{12}$$

The other versions can be figured based on the *pure minor*.

Section 10

Keys
Major

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Scales and *keys* are almost synonymous. A *key* is basically the collection of *notes* included in the *scale*. That *key* is identified by the *tonal center* of the *scale*. So, if the *scale* is an E♭ major *scale*, the *key* is E♭ major.

Before we get too far into this topic, let's look at some new terms. Each *scale degree* actually has a name, beyond its corresponding number. These names often identify a specific *function* and will serve us on down the road as we discuss *diatonic harmony*. For now, let's just learn the *scale degree* names and what they represent.

Since these apply to all *keys*, we'll just use the *scale degree* numbers instead of specific printed musical *notation*. Here's a simple chart with the *scale degree* numbers and the corresponding names to get us started.

♬

Scale Degree	$\hat{1}$	$\hat{2}$	$\hat{3}$	$\hat{4}$	$\hat{5}$	$\hat{6}$	$\hat{7}$
Name	Tonic	Supertonic	Mediant	Subdominant	Dominant	Submediant	Leading tone

Let's look at each of these and try to give some meaning to the labels. They are not listed below in numeric order. The first three play significant roles in the construction of *diatonic harmony* and the others relate to them, as you'll see.

Here are three terms that will help clarify the meaning of some of the names.
super = above; sub = below; mediant = middle (halfway)

Tonic - the *tone* on which the *scale* and the *key* are based (*tonal center*)

Dominant - has the strongest influence in the *functions* of *tonal harmony;* a P5 above the *Tonic*

Subdominant - another strong influence in the *functions* of *tonal harmony*; a P5 below the *Tonic*

Supertonic - the *tone* immediately "above" the *tonal center*

Mediant - the *tone* that is halfway to the *Dominant,* up from *Tonic*

Submediant - the *tone* that is halfway to the *Subdominant,* down from *Tonic*

Leading tone - the *tone* that "leads up to" or "leans up toward" the *Tonic* (see discussion in Section 9)

Let' rearrange the chart to demonstrate the labeling.

Scale Degree	$\hat{4}$	$\hat{6}$	$\hat{7}$	$\hat{1}$	$\hat{2}$	$\hat{3}$	$\hat{5}$
Name	Subdominant	Submediant	Leading Tone	Tonic	Supertonic	Mediant	Dominant

Let's get back to *keys*. As you already know, every *major scale* will have the same pattern of *whole-steps* and *half-steps*. As *tonal centers* change, so will the number of *accidentals* in the music, which seems pretty simple and uncluttered if you're only dealing with one or two.

What if you have a piece of music in which the *tonal center* is G♭? It might look something like the excerpt on the following page.

As a composer, that's a whole lot of *flats* to have to write into your music. As a performer, this *score* would seem a bit too cluttered.

♫ This is one of the basic reasons for having ***key signatures***. A *key signature* simply puts all of the *accidentals*, required for the particular *scale* or *key* being used, at the beginning of each *staff*. With a *key signature* for G♭ major, the piece above would look like this:

We should already be very familiar with the key of *C major*. As I mentioned before, it's the most accessible, since it's *key signature* has no *accidentals*. We've also been introduced to *F major* and *G major*. Those are the *keys* that only have one *accidental* in the *signature*. *F* has one *flat* and *G* has one *sharp*.

As I'm thinking about these *keys*, the topic of relationships comes to mind. These two *keys* are considered *closely related* to *C*. If you remember the DNA metaphor associated with *relative major/minor scales*, this is only one step removed from that. Both the *keys* of *F* and *G* have six of their seven *pitches* in common with *C*. There is only one *pitch* that separates each of them from *C*. But… as fascinating as all of this might seem, we need to wait and talk more about these family trees later. For now, let's focus on figuring out the rest of the *key signatures*.

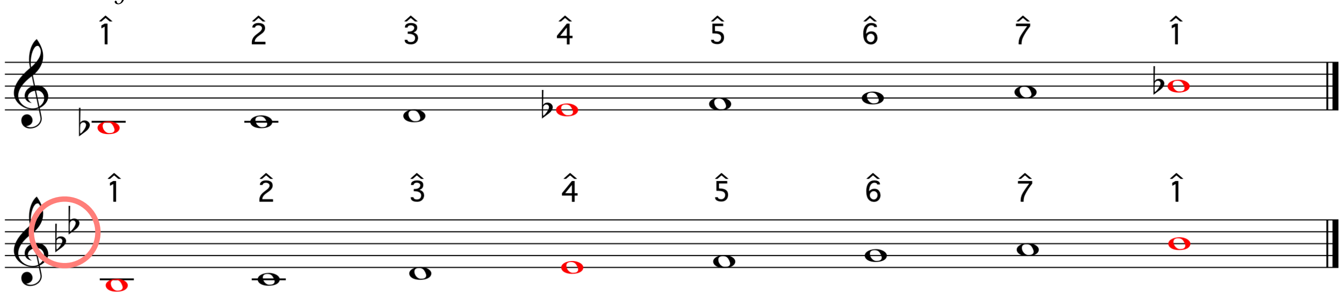
How do we know how many *accidentals* to put in the *key signature*? How do we know what order to put them on the staff? Those are certainly good, and very common questions. We already know that the *tonal center* and our *whole/half-step* pattern will determine where the *accidentals* will show up in the *scale*. Now, we just need a method to put all of them into a specific order.

♫ Let's use *C major*, *F major*, and *G major* as our starting points. From *C major* we added one *flat* and got *F major*. What's the relationship between C and F? F is a P5 down from C. So, we go down a P5 and that adds 1 *flat* to the *key signature*. The graphic on the next page shows the *F major scale* with B♭ written in, then with B♭ in the *key signature*.

F major

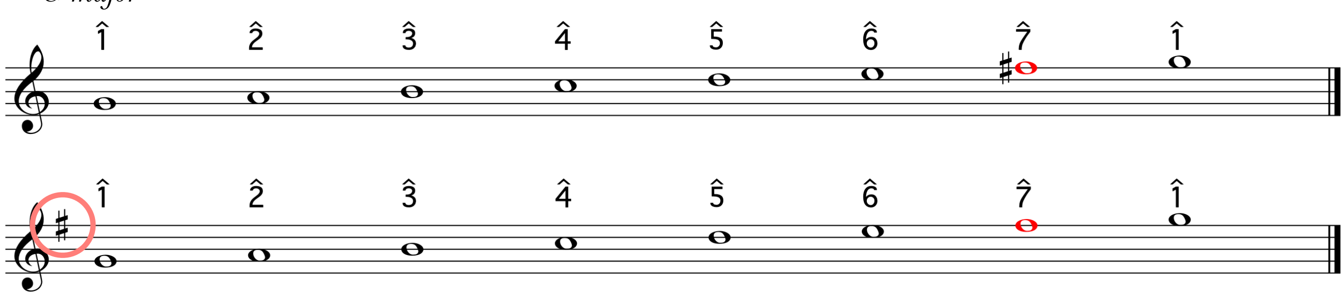
Going down another P5 will take us to B♭ with 2 *flats* in the *signature*.

B♭ major



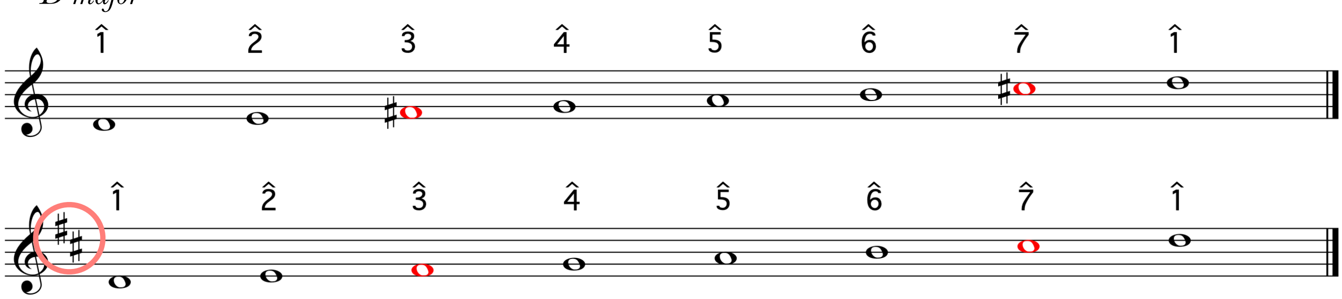
♫ This works the same way with *sharps*, but in the opposite direction. If we go up a P5 from C, it takes us to G with a *signature* of 1 *sharp*.

G major



…up another P5 takes us to D and 2 *sharps*.

D major

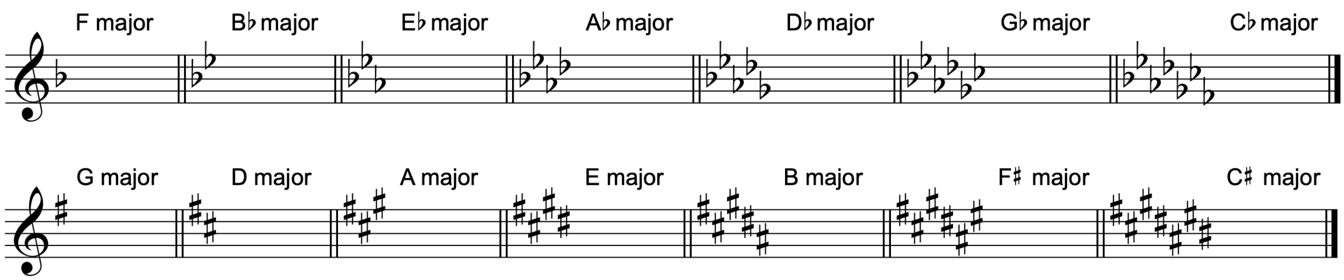


It seems there is a logical yellow brick road for us to follow.

Whichever direction we're headed, going another P5 in that direction adds another *accidental* to the *key*.

Whether we're going up or down, this hike through the *key signatures* will come to the end of the trail when we get to 7 *flats* or 7 *sharps*.

♫ As usual, here are more graphics to help us visualize the process.



If you're thinking this through, you may be wondering, "Why does this stop at 7? Aren't there 12 *pitches* in the *octave* and on the the CPS? What happens if you keep going up or down a P5?"

Those are very good questions. …very perceptive. I'm glad you chased that rabbit, because it gives us an opportunity to see just how interconnected and overlapping all of this is.

If we keep going in the direction we started it will eventually take us to the other side (*flats* to *sharps* or *sharps* to *flats*) and ultimately back to *C major*. If we started down the path with *flats*, at one point we'll flip over to the *sharp* side and go back the other direction. I know… that's a bit confusing.

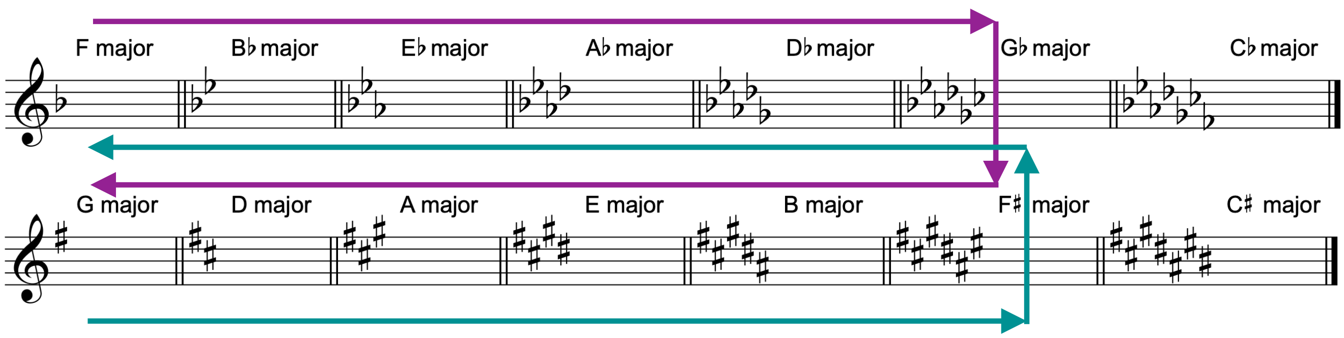
Think about the *tritone*. Remember, it cuts our CPS in half. It's the exact middle of the *octave*. If we're following a progression of P5s, with C as our starting point, going one direction will bring us to G♭. Going the other direction will bring us to F♯. Those are *enharmonically* the same. …the *tritone* away from C. …six *half-steps* from our starting point. There are six *half-steps* left.

If we're determined to follow the *key signature* path and stick with our original *accidentals* (*flats* or *sharps*), there's only one more step. …G♭ to C♭ or F♯ to C♯. That's it. …no more *key signatures*.

BUT… What if we want to keep going with our P5 progression? The *tritone* could be our portal to the other side. In this trek, when we get to the *tritone*, we can switch from one set of *accidentals* to the other. We could switch from G♭ to F♯ (or vice-versa) and keep going with our progression of P5s.

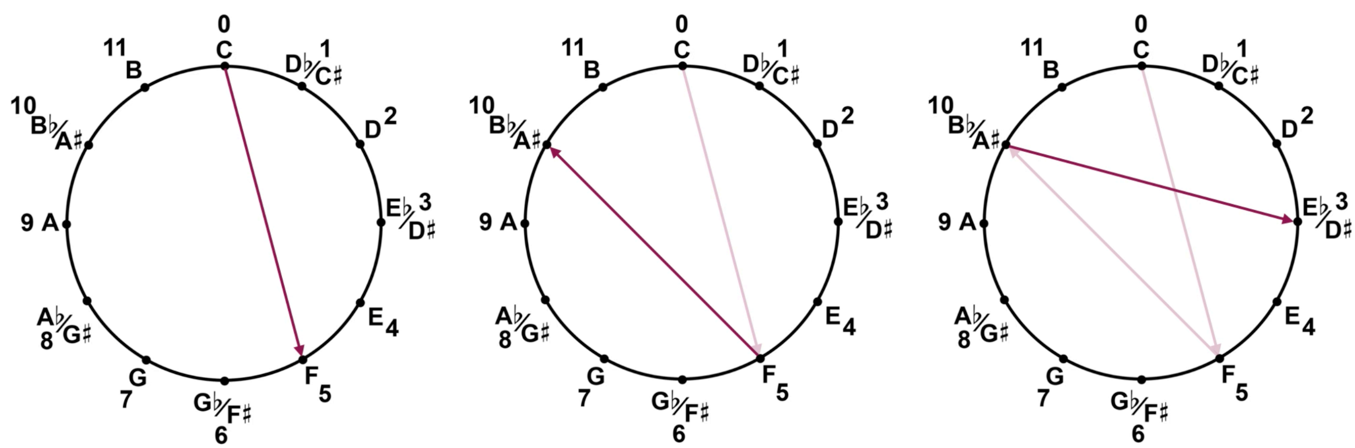
If we have followed the *flat* path down to G♭, switch to the enharmonic F♯, then keep progressing down by P5s, we'll be following the *sharp* path backwards to C. Of course, the opposite would be true as well. …forward through the *sharps*, switch at the *tritone*, then backward through the *flats*.

We need a graphic for this. On the next page, see the *key signature* chart with colored lines and arrows to help you visualize the path the P5s will take. Follow the plum color for the *descending* path and the teal color for the *ascending* path.

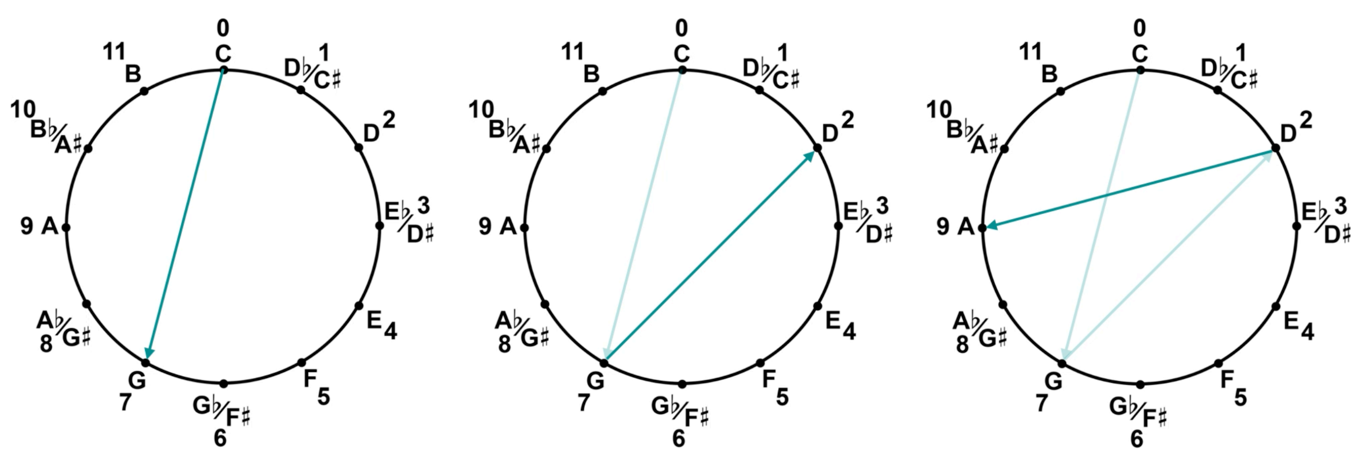


Let's see how this progression looks on the CPS (just the first three in the sequence). Can you fill in the rest?

Descending



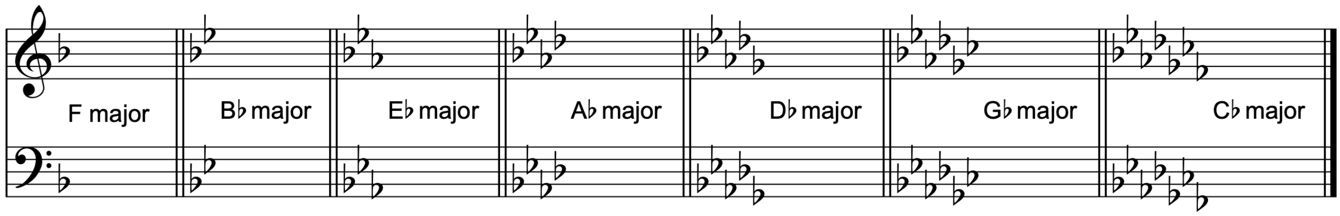
Ascending

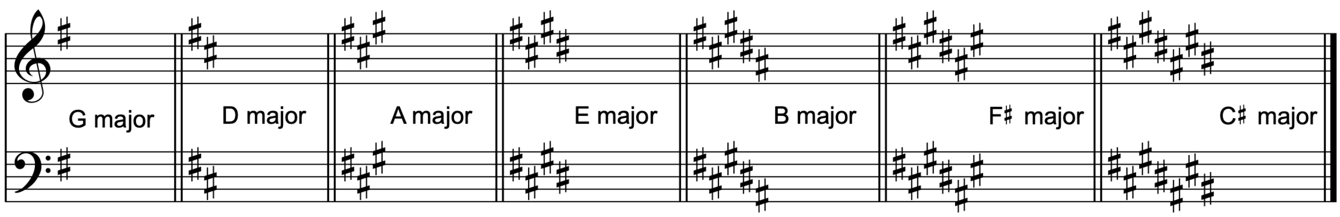


This progression is generally referred to as the *Circle of 5ths*. Our study presents it a bit differently than the traditional approach.

Unfortunately, static pictures can't quite capture the whole progression very effectively. The e-book version of this actually has video examples of the whole progression. Those can be found here.

♬ Here's a quick note. The *accidentals* are always put on the *staff* (both *treble* and *bass clefs*) in the same order and in the same place on the *staff*. Please pay close attention to the charts below in order to remember that placement.





Now that we understand *key signatures* and know how to put the *accidentals* on the *staff*, can we remember, by just looking at a *key signature*, which *key* it is? Of course we can. We can just commit them to memory. As musicians, we'll be dealing with these on a regular basis and should know them instinctively. In reality, though, most of the music we'll regularly see will probably only include about seven or eight keys. Here are a few practical tips to help remember them all.

♬ With the *flat keys*, the next to the last *flat* in the *key signature* is the *key*. For example, if there are five *flats* in the *key signature*, the fourth *flat* will indicate the *key*. The fourth *flat* to be added is D♭, so five *flats* is the *key* of *D♭ major*. This works for all the *flat keys* except *F major*. We'll just have to memorize that one.

♬ With the *sharp keys*, the last *sharp* in the *key signature* will be the *leading-tone* of the *key*. So, the *key* will be one *half-step* up from the last *sharp*. For example, if the last *sharp* in the *key signature* is D♯, then the *key* will be *E major*.

♬ If the *key signature* has six *accidentals*, that *key* is a *tritone* away from C. If there are six *flats*, the *key* is *G♭ major*. If there are six *sharps*, the key is *F♯ major*.

♬ For the two *key signatures* with all seven *accidentals*, take C and add the *accidental* to the name. Seven *flats* will be *C♭ major*. Seven *sharps* will be *C♯ major*.

I think those are all the practical tips I have up my sleeve. Let's move on to see how these *key signatures* relate to the *minor scales*.

Section 11

Keys
Minor

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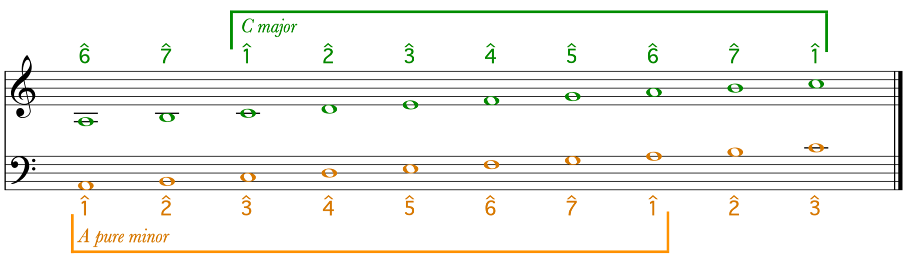
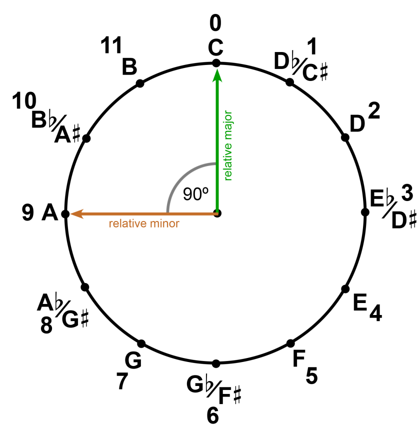
If you remember our discussion of the [*Pure/Natural Minor scale*](), you'll recall that we referred to the *pure A minor scale* as being the *relative minor* of *C major*. We also noted that a *pure/natural minor scale*, built on the $\hat{6}$ of any *major scale*, would have exactly the same *pitches* as that *major scale*. The difference is the *tonal center*. For example, *C major* and *A natural minor* have exactly the same *pitches*, but *C major* has C as its *tonal center* and *A natural minor* has A as its *tonal center*. These *scales* illustrate the *relative major/minor scale* relationship. ([see the notes and CPS graphs on page 43]())

♫ Understanding this relationship between the *major scale* and the *relative minor scale* should make our discussion of *minor keys* so much easier. The fact is, a *minor scale* has the same *key signature* as its *relative major scale*.

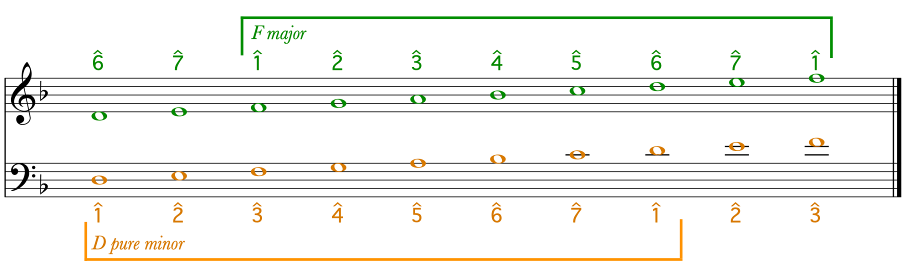
♫ So, you might ask, "How do we know what the *relative major* is?" If we think about the *relative major* and *minor scales* as overlapping each other, the *minor* begins and ends on the $\hat{6}$ of the *major*. That being the case, the *major* would begin and end on the $\hat{3}$ of the *minor*.

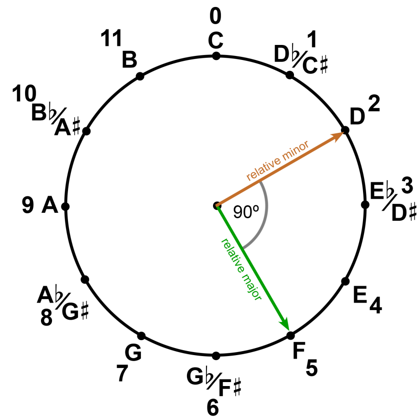
Once again, I think a picture can save a few words. Below are musical examples along with the CPS showing that the *relative major/minor scales* and *keys* are always at a 90º angle (more later).

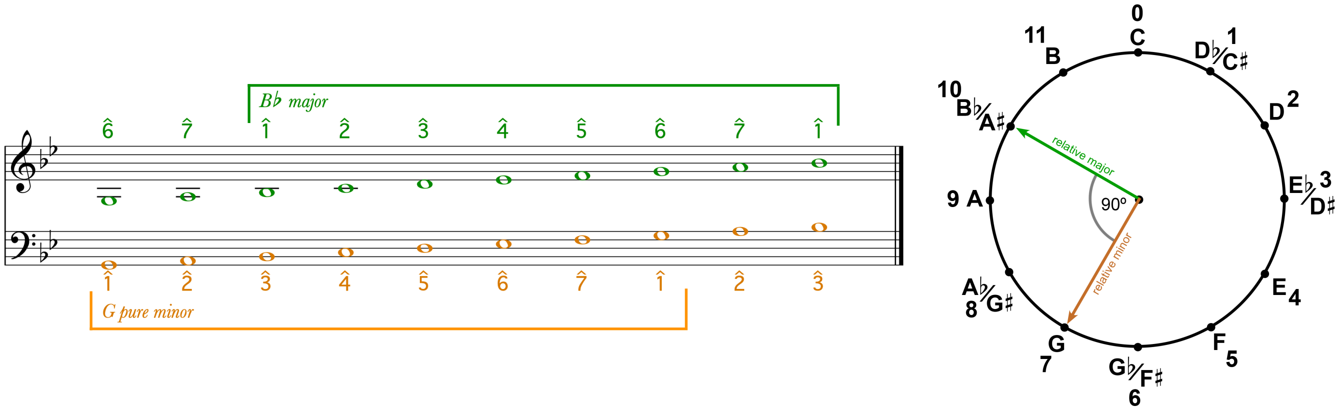


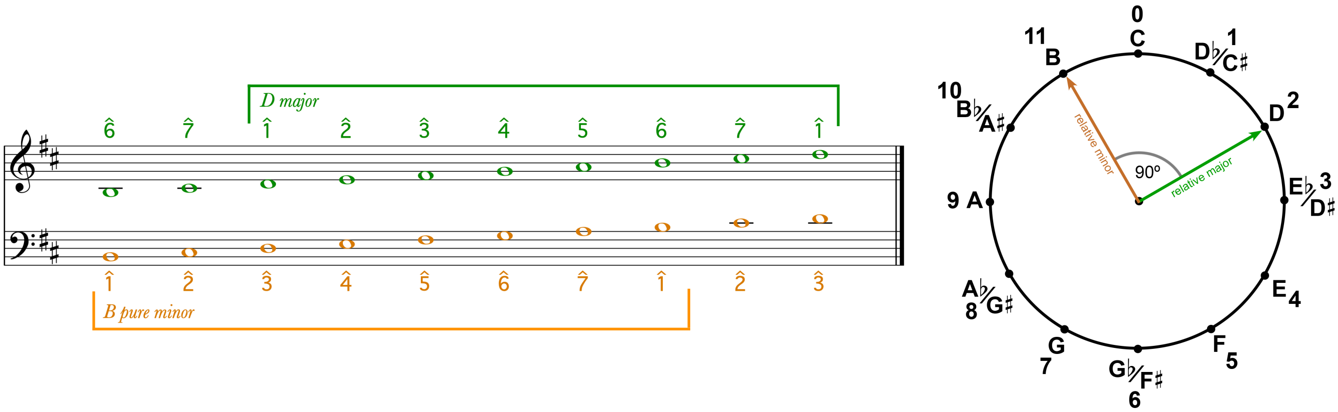


Notice that each *scale* has its own *scale degrees*, based on its own *tonal center*.









Let's rewind for a moment and take a Mathematical Mystery Tour (Beatles reference) to see what's happening on our CPS. For each of the *scales* above, the CPS shows a 90º (ninety degree) relationship between the *major* and the *relative minor* (and vice-versa). Why is it 90?

If you were to stand on the middle dot of the CPS, facing 0 (C), then turn to face each of the points on the circle, you would ultimately return to 0. You would have turned 360º to come back to your original starting point. Since the circle has 12 points along the tour, 360 would be divided by 12. Each stop of the tour would be 30º away from the previous point and 30º on to the next (12 of those equaling 360º).

Following this logic, each *half-step* of the CPS will equal 30º. So, each *interval* will have an equivalent angle, measured in degrees. Since the *major* and *relative minor* are a m3 (3 *half-steps*) apart, 30º times 3 equals 90º. To see the degree equivalents for each of the *intervals* and *inversions*, view the included video here (e-book version only).

♫ Getting back to our *minor key signatures*… The *key signature* for any *minor scale* will be the same as its *relative major*, which is a m3 above the *minor scale tonal center*.

If you're trying to put all the pieces in this puzzle, you're probably wondering about the other two versions of the *minor scale*. Will there be different *key signatures* for the others, since they have additional *accidentals*? The quick but definitive answer is "no." The *accidentals* needed for the *Harmonic Minor* and *Melodic Minor scales* will simply be added to the music wherever they are necessary. Those will not be added to the *key signature*.

As a taste of things to come, the *accidentals* used in the *minor scale* will be one of the signs you'll look for when analyzing a piece of music, trying to determine the *tonal center*. …more fun in store.

Section 12

Rhythm
Note Values

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The most direct and obvious connection between math and music comes to light in the study of *note values* and *rhythm*. *Note values* are simply the building blocks for the element of *rhythm*. We could say that *rhythm* is the linear combination of proportional values in time. Wow! What does that really mean?

Well… we need to remember that music only exists in time. We can't experience a piece of music like we do a painting or sculpture. A whole piece of music does not exist at a single fixed point in time. It takes a length of time to unfold and be heard. ♫ *Rhythm* is how time is divided up in a piece of music.

Notes can resonate for a long period of time or they can pass by in quick succession. There can be continuous sound or moments of silence. It's those combinations of long and short, sound and silence, stopping and starting, that make a piece of music come to life.

There's still that phrase, "proportional values." What are those?

As with most musical elements, *rhythm* exists in a context of relativism. Each *rhythmic value* takes on a function relative to its surroundings, whether that's the *meter* it's in (more on this later) or the other values with which it functions. Even within a context of relativism, the numeric and proportional relationships between the *note values* remain constant. ♫ *Note values* function like fractions. They will always be in specific proportion to the other values.

The chart immediately below illustrates each note's symbol, value, name, and the equivalent rest (silence) symbol.

NOTE	VALUE	NAME	REST
𝅝	1	Whole	𝄻
𝅗𝅥	1/2	Half	𝄼
♩	1/4	Quarter	𝄽
♪	1/8	Eighth	𝄾
𝅘𝅥𝅯	1/16	Sixteenth	𝄿
𝅘𝅥𝅰	1/32	Thirty-second	𝅀

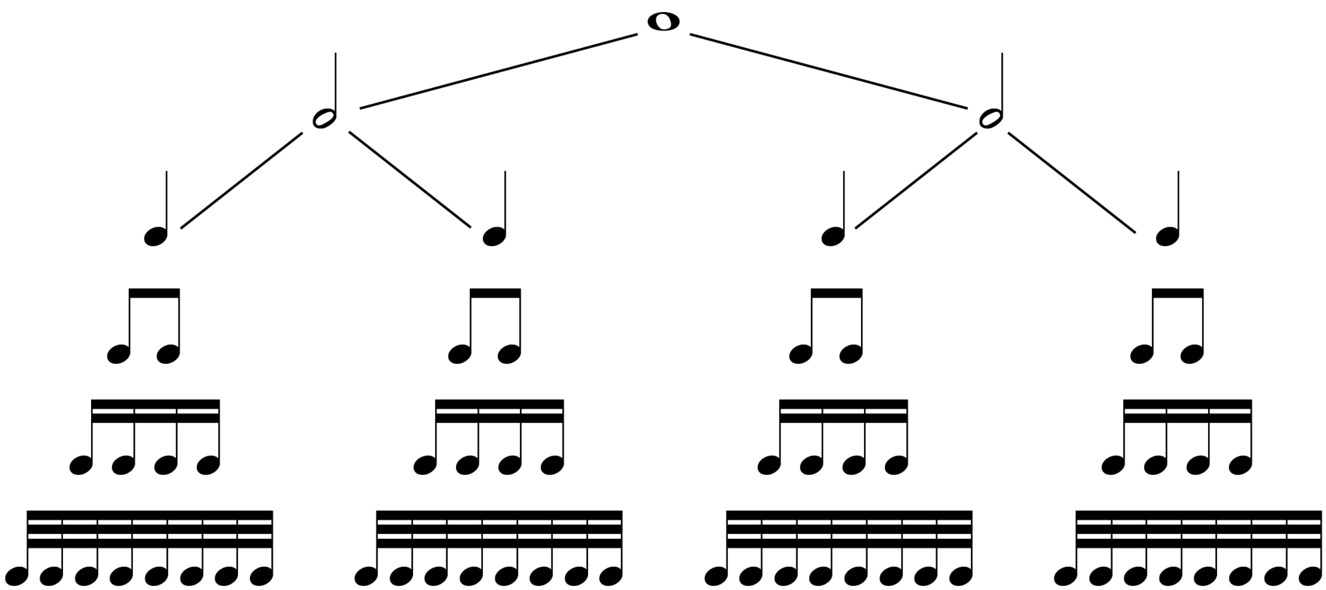
These values and proportional relationships will remain constant within any context.

Looking at the bigger picture on the next page, we begin to see, in musical notation, the equivalences.

The *whole note* is equivalent to two *half notes*. Each *half note* is equivalent to two *quarter notes*, and so on. It is also true to say that one *whole note* is equivalent to four *quarter notes*, and one *half note* is equivalent to four *eighth notes*, etc.

Let's look at it this way:

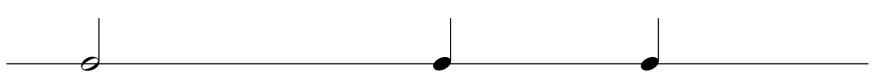
1 *whole note* = 2 *half notes* = 4 *quarter notes* = 8 *eighth notes* = 16 *sixteenth notes* = 32 *thirty-second notes*



The combinations of these *note values* that create *rhythm* in a piece of music may not be infinite, but there have been more than enough to keep composers busy for centuries. We certainly can't scratch the surface of the possibilities, but let's look at a few just to familiarize ourselves with the way they work… and let's add the mathematical equations to give us a broader perspective on the relationships.

Let's talk about math for a moment. We are assuming that you are already familiar with fractions and how they work. We will be adding and multiplying fractions as we deal with *rhythm*. If you need a basic refresher click here (e-book only). This website is practical and straightforward. It might help.

For now, let's bundle some combinations into <u>groups that will equal one *whole note* (1)</u>. Here are some examples:



$$\frac{1}{2} \quad + \quad \frac{1}{4} \quad + \quad \frac{1}{4} \quad = \quad 1$$

Let's bracket some of these together to group them into sets.

$$\left\{\frac{1}{2}\right\} \quad + \quad \left\{2\left(\frac{1}{4}\right)\right\} \quad = \quad 1$$

$$\frac{1}{2} \quad + \quad \frac{1}{2} \quad = \quad 1$$

Notice the two sets. The first set just contains a *half-note* {1/2}. The second set has two *quarter notes* {2(1/4)}. If you'll remember from your basic math studies, the group in parentheses will be multiplied by the number that precedes it. So, 2 X 1/4 = 2/4, and 2/4 can be reduced to 1/2. That will give us 1/2 + 1/2. When you add those two sets together you get 2/2 which reduces to 1 (*a whole note*).

Let's look at a few more. This time we'll group them into sets (differentiated by color), but won't verbalize them (that will be the task you do in your head).

$$\tfrac{1}{4} + \tfrac{1}{4} + \tfrac{1}{8} + \tfrac{1}{8} + \tfrac{1}{8} + \tfrac{1}{8} = 1$$
$$\{2(\tfrac{1}{4})\} + \{4(\tfrac{1}{8})\} = 1$$
$$\tfrac{1}{2} + \tfrac{1}{2} = 1$$

$$\tfrac{1}{4} + \tfrac{1}{8} + \tfrac{1}{8} + \tfrac{1}{16} + \tfrac{1}{16} + \tfrac{1}{16} + \tfrac{1}{16} + \tfrac{1}{4} = 1$$
$$\{\tfrac{1}{4}\} + \{2(\tfrac{1}{8})\} + \{4(\tfrac{1}{16})\} + \{\tfrac{1}{4}\} = 1$$
$$\tfrac{1}{4} + \tfrac{1}{4} + \tfrac{1}{4} + \tfrac{1}{4} = 1$$

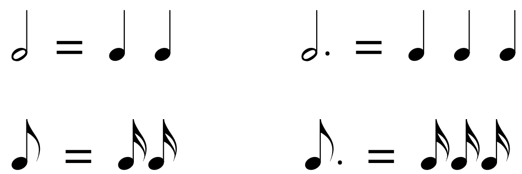
$$\tfrac{1}{2} + \tfrac{1}{16} + \tfrac{1}{16} + \tfrac{1}{16} + \tfrac{1}{16} + \tfrac{1}{8} + \tfrac{1}{8} = 1$$
$$\{\tfrac{1}{2}\} + \{4(\tfrac{1}{16})\} + \{2(\tfrac{1}{8})\} = 1$$
$$\tfrac{1}{2} + \tfrac{1}{4} + \tfrac{1}{4} = 1$$
$$\{\tfrac{1}{2}\} + \{2(\tfrac{1}{4})\} = 1$$
$$\tfrac{1}{2} + \tfrac{1}{2} = 1$$

Did you notice how we grouped, calculated the equivalents, and grouped again in order to arrive at equal *note values*? This allowed us to easily combine those equal values to total up to the *whole*? This process is not absolutely necessary, but it is one way of demonstrating a logical path for identifying and calculating *note values* using the corresponding math functions.

You've probably noticed that up to this point all of our *note values* are divisible by 2 and 4. You may be wondering if that's always the case or if they are ever in subdivisions of 3 or 6.

It's time to introduce the *dot* (•). The *dot* added to any *note value* adds one half of the *note's* value to it. For example, a *quarter note* is equal to two *eighth notes*, and half of that will be one *eighth note*. So, a *quarter* with a *dot* (*dotted quarter*) will equal 3 *eighth notes*. Here's the concept in a visual form.

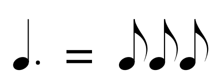
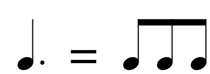
How about two more examples?

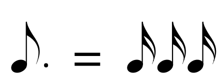
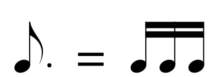


♫ Let's stop the bus for a moment and talk about the different parts of the *note*. You may or may not be aware, there are three parts to a musical *note*.

1) There's the <u>*note head*</u>, which can be open (𝅗) or closed (●).
2) Unless the note is a *whole note*, it will have a <u>*stem*</u> (|) connected to the *note head*.
3) If the *note value* is less than a *quarter note* it will have at least one *flag* (𝅮) connected to the *stem*.
   One flag indicates an *eighth note* (♪). Two flags… a *sixteenth* (𝅘𝅥𝅯). …and so on.

Let's get back on the bus and continue talking about *note values*. The reason we stopped was to make sure you understood basic *note* construction before we talk about options. As we write musical *notation*, it is sometimes favorable to group *notes* together into units. These units help us keep track of equivalent values. The way this is done is to *beam notes* together. If there are several *notes* together, with values less than a *quarter*, the *flags* can be converted to *beams*. Let's take the examples immediately above to demonstrate this.

If we have this relationship:  It could be rewritten like this: 
Beaming the three *eighth notes* together should make it easier to view them as a unit equal to the *dotted quarter*.

Here's one more example:  …rewritten as: 

If you'll go back two pages and look at our *note value* equivalency graph, you'll notice how the units are *beamed* together. This makes it easier to see how one group is equal to another.

Speaking of groups… the next three sections will show us traditional approaches to grouping our *notes* together. One will use *subdivisions* of 2 and 4. One will use 3 and 6, and the last will use a combination of both. Good times!

Section 13

Rhythm
Simple Meter

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In most music, the *rhythmic* combinations are organized into groups that all have the same number of *beats*. These groups are called *measures.* (Some folks will call these *bars.* …not to be confused with drinking establishments).

So… What is a *beat*? A *beat* is simply the *note value* that serves as the dividing/organizing unit of the *measure*. Each and every *beat* in a *measure* will have the same equivalent value. (This definition will apply until we talk about *asymmetrical measures* in which the *beats* have unequal values. That's for another day.)

How our *rhythmic* groupings are measured, or metered, is what we're discussing. The next concept/term to consider is *meter*. The *meter* of a piece of music is what tells us how our *measures* are organized. Looking at some actual *meters* will help clarify all of this.

In this section we are discussing ***simple meter***. In each of these *meters*, the *beats* will have subdivisions of 2, 4, 8, etc.

There are two numbers used to identify *meter*. They look similar to fractions but have a different function. *Meter* designations (*time signatures*) will look like this:

$$\begin{matrix}4\\4\end{matrix} \qquad \begin{matrix}3\\2\end{matrix} \qquad \begin{matrix}2\\8\end{matrix}$$

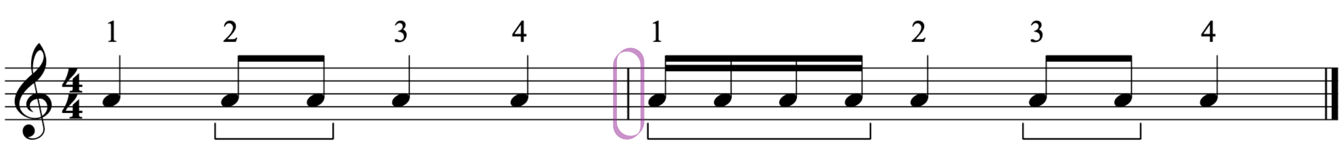
♫ The bottom number of each set indicates which *note value* is the *beat*. The top number tells us how many *beats* are in one *measure*. Let's look at the *meters* noted above in a little more detail to see how this works.

$\begin{matrix}4\\4\end{matrix}$ $\begin{matrix}= \text{four } \textit{beats} \text{ in a } \textit{measure}\\= \text{the } \textit{quarter note} \text{ is one } \textit{beat}\end{matrix}$

$\begin{matrix}3\\2\end{matrix}$ $\begin{matrix}= \text{three } \textit{beats} \text{ in a } \textit{measure}\\= \text{the } \textit{half note} \text{ is one } \textit{beat}\end{matrix}$

$\begin{matrix}2\\8\end{matrix}$ $\begin{matrix}= \text{two } \textit{beats} \text{ in a } \textit{measure}\\= \text{the } \textit{eighth note} \text{ is one } \textit{beat}\end{matrix}$

Here are those same *meter* examples illustrated on a *staff*. Note the vertical lines (circled) on the *staff* that separate the *measures*. Those are generally referred to as *bar lines* (see the first paragraph on this page).



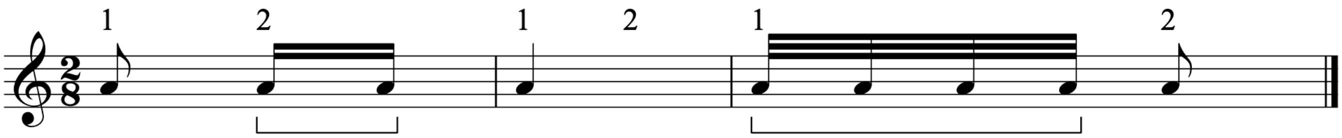
$\frac{1}{4}$ $\{2(\frac{1}{8})\}$ $\frac{1}{4}$ $\frac{1}{4}$ $\{4(\frac{1}{16})\}$ $\frac{1}{4}$ $\{2(\frac{1}{8})\}$ $\frac{1}{4}$

$\frac{1}{4}$ $\frac{1}{4}$ $\frac{1}{4}$ $\frac{1}{4}$ $\frac{1}{4}$ $\frac{1}{4}$ $\frac{1}{4}$ $\frac{1}{4}$

Notice how the *note* groupings are all equivalent to a *quarter note* and that each *measure* has the equivalent of four *quarter notes*.

$\frac{1}{2}$ $\{2(\frac{1}{4})\}$ $\{4(\frac{1}{4})\}$ $\{2(\frac{1}{4})\}$ $\frac{1}{2}$ $\frac{1}{2}$

$\frac{1}{2}$ $\frac{1}{2}$ $\frac{1}{2}$ $\frac{1}{2}$ $\frac{1}{2}$ $\frac{1}{2}$

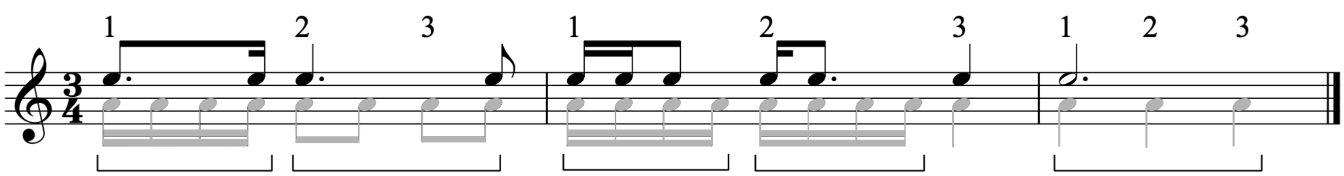


$\frac{1}{8}$ $\{2(\frac{1}{16})\}$ $\{(\frac{1}{4})\div 2\}$ $\{4(1/32)\}$ $\frac{1}{8}$

$\frac{1}{8}$ $\frac{1}{8}$ $\frac{1}{8} + \frac{1}{8}$ $\frac{1}{8}$ $\frac{1}{8}$

The example of the **3/2** *meter* is very straightforward. It's easy to identify each equivalent group. The **2/8** has something new. Notice what happened in the second *measure* of that example. The *quarter note* actually occupied the space of two *beats*. Most *note* groupings will be added together to be the equivalent of a *beat*. Other *note values* will be the equivalent of more than one *beat*.

Let's try an example that utilizes *dotted note values*. The *notes* in gray are just there to indicate the *subdivisions*. The colors for the equations are there to help us keep the *measures* separate.



$\{(3/16)+(1/16)\} + \{(3/8)+(1/8)\}$ $\{(2/16)+(1/8)\} + \{(1/16)+(3/16)\}+1/4$ $3/4$

$4/16 + 4/8$ $4/16 + 4/16 + 1/4$ $3/4$

$1/4 + 2/4$ $1/4 + 1/4 + 1/4$ $3/4$

$3/4$ $3/4$ $3/4$

This example is a bit trickier. Notice in the first *measure*, the *dotted quarter* is the equivalent of a *beat* and a half. The *eighth note* at the end of that *measure*, added to the *dotted quarter*, makes a two-*beat* grouping. Likewise, the *dotted half* in the last *measure* makes a three-*beat* group by itself.

If you follow the equations in this example, line by line, you should be able to see how each *measure* adds up to the equivalent of three *quarter notes* $\{3(\frac{1}{4})\}$ in every *measure*, just like the *time signature* indicates.

There's one other item to take note of. For the *dotted rhythms*, the fraction we used was based on the *subdivision* values. The *dotted eighths* in the first two *measures* are identified as **3/16**. The *dotted quarter* in the first *measure* is **3/8**, and the *dotted half* in the last *measure* is **3/4**. This concept will be significant as we explore the world of *compound meter* in the next section.

Section 14

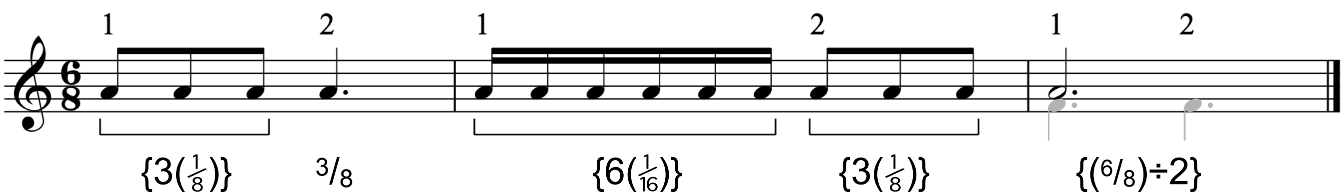
Rhythm
Compound Meter

Table of Contents

Music theory is one of those subjects that is cumulative. Everything we learn is built on everything we've previously learned. I hope, at this point in our study, we all have a solid foundation of *rhythmic values*, their relationships, and their organization. As with any construction project, the new material just keeps being delivered and it's up to us to utilize it to strengthen and expand what's already in place. *Compound meter* is new material. It's an integral part of the structure. It will add a new dimension to our existing foundation. So, let's buckle up our tool belts and get this nailed down.

As was alluded to back on page 61, *simple meter* is based on the *duple subdivisions* of 2, 4, 8, etc. while *compound meter* is based on the *triple subdivisions* of 3, 6, 12, etc. As you remember, these *triple subdivisions* occur in *dotted note values*.

♫ In *compound meter*, the *beat* is always a *dotted note value*. Let's look at one of the most common *compound meters* to see how all of this is constructed.



If we look at the *time signature* above, with only an understanding of *simple meter*, we would think that the *eighth note* is the *beat* and there are 6 *beats* in the *measure*. That is not the case. Remember, in *compound meter* the *beat* will be a *dotted rhythmic value*. Notice, in the example above, how the *eighth notes* are bracketed together. Those groups of 3 *eighth notes* represent 1 *beat*. 1 *beat* = $^3/_8$, which is the equivalent of 1 *dotted quarter*. If the *dotted quarter* equals 1 *beat*, how many *beats* are in each *measure*?

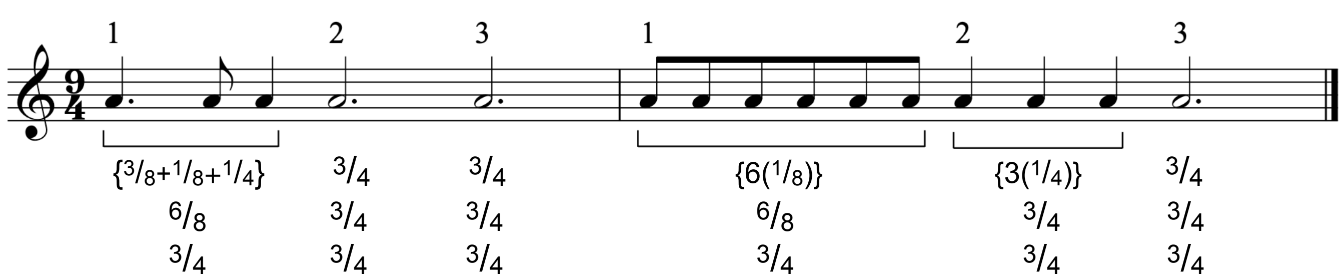
If there are 6 *eighth notes* in a *measure* and $^3/_8$ is the *beat*, then… $\{6(\frac{1}{8})\} \div {}^3/_8 = 2$

Another way to conceive it is… How many $^3/_8$s are in $^6/_8$?

♫ When looking at a *time signature*, if the top number is a multiple of 3, then it will take 3 of the *note value* indicated with the lower number to equal 1 *beat*. To determine how many *beats* will be in one *measure*, we can take the top number and divide it by 3. Here's how that breaks down for **6/8**.

6	$\rightarrow (6 \div 3) =$	2	beats in the measure
8	$\rightarrow \{3(\frac{1}{8})\} =$	$^3/_8$	dotted quarter beat

Below is an example that will have the *dotted half* as the *beat*. Remember, a *dotted half* will be mathematically identified as $^3/_4$ (3 *quarter notes*). This *meter* will have 3 *beats* with the *dotted half* as the *beat*.



Just for clarity, let's look at what's in our toolbox for a moment.

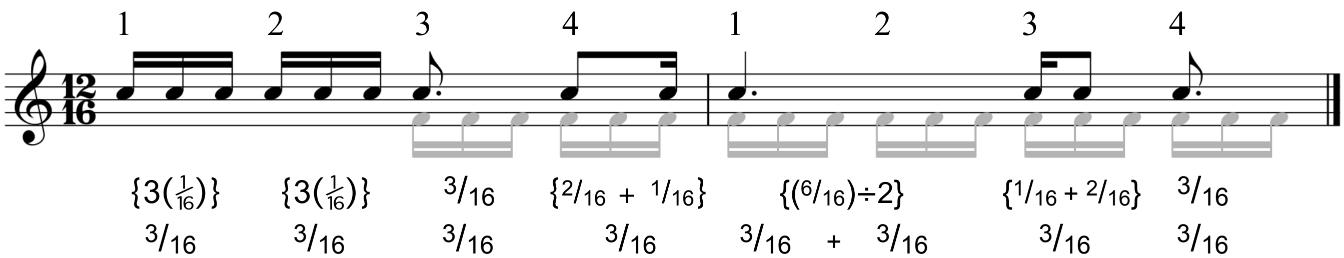
If we have 3 *eighth notes*, those will be designated as a set and will look like this: $\{3(\frac{1}{8})\}$
That will be calculated as $3 \times \frac{1}{8}$. The equivalent will be $^3/_8$.

Even though they are equivalents, this set is not to be confused with a *dotted quarter,* which will be designated as a single fraction: $^3/_8$

Here's how that previous *meter* breaks down.

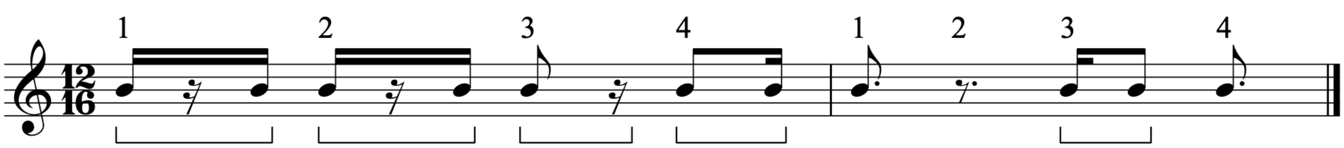
9 $\rightarrow (9 \div 3) =$ 3 beats in the measure
4 $\rightarrow \{3(\frac{1}{4})\} =$ $^3/_4$ dotted half beat

Here's one more for good measure (pun intended). The *notes* in gray are just to show the *subdivisions*.



12 $\rightarrow (12 \div 3) =$ 4 beats in the measure
16 $\rightarrow \{3(^1/_{16})\} =$ $^3/_{16}$ dotted eighth beat

♫ Before we move on, we need to be reminded that any *note value* has an equal *rest value* which can occupy the same space. Here's one example. …a variation of the one immediately above:



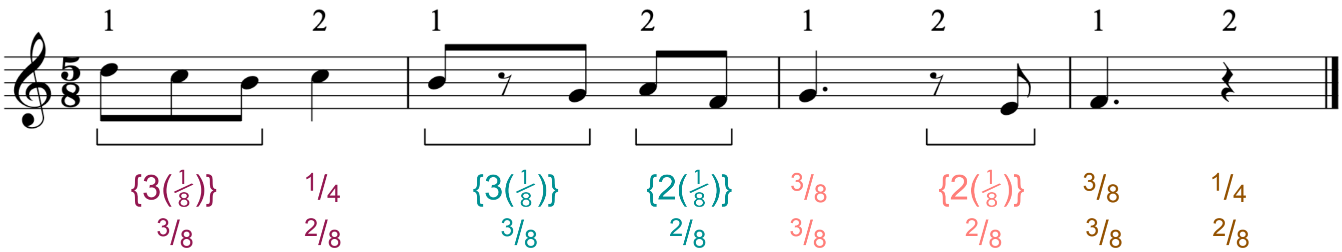
Section 15

Rhythm
Asymmetrical Meter

Have you ever gone to a restaurant, sat down in a chair, and realized that one of the legs wasn't the same length as the others? You found yourself feeling a bit unstable and the chair rocked a bit when you would lean a certain way. Well… that's a bit like *asymmetrical meter*.

♫ In *asymmetrical meter*, at least one of the *beats* has a different number of *subdivisions* than the others. It's really just a combination of *simple* and *compound* beats. Some of the *beats* in the *measure* have *duple subdivisions* and some have *triple subdivisions*.

Let's take a quick look at what is probably the simplest *asymmetrical meter*, **5/8**.

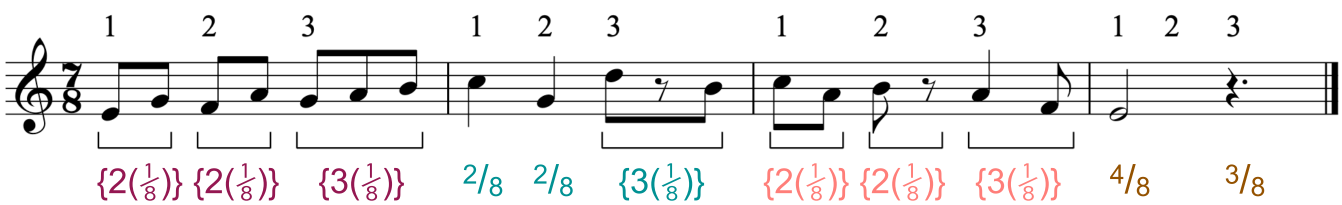


As you can see, the first *beat* of each *measure* is what we would consider *compound time* (*triple subdivision*) and the second *beat* of every *measure* is *simple time* (*duple subdivision*). This combination is what gives this example of **5/8** the label of *asymmetrical*. It is unbalanced.

Unless the *tempo* of the piece is very slow, this would be *conducted* with two *beats* per *measure*. Of course, the *beats* are unequal. Let's break it down.

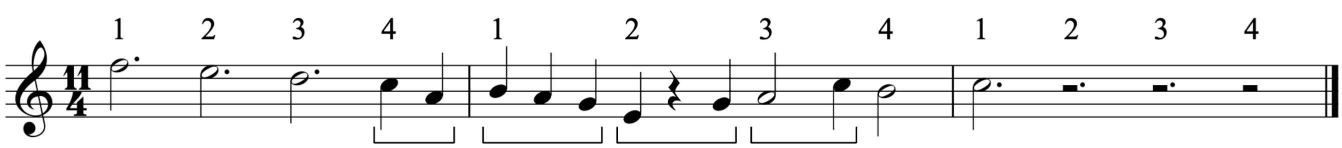
5 → {(3) + (2)} = 2 beats in the measure
8 → {3(1/8) + 2(1/8)} = 3/8 (dotted quarter beat) + 2/8 (quarter beat)

Here's another example. Let's break it down just in eighth notes.



It should be easy to see that there are 3 *beats* in each *measure*. The first two are *simple time* (2/8) and the third is *compound* (3/8). So… 2 + 2 + 3 = 7

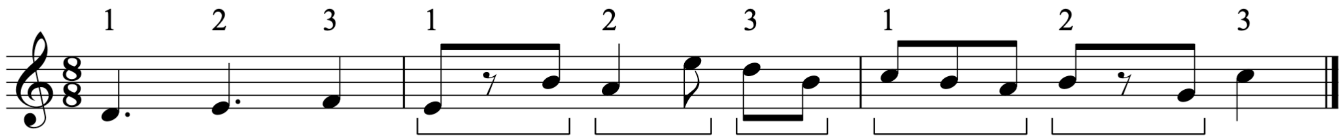
Of course, not all *asymmetrical meters* will have the *eighth note* as the common denominator/*note value*.



We can see here that there are 4 *beats* in each measure. The first three are *compound time* (3/4) and the last is *simple time* (2/4). 3 + 3 + 3 + 2 = 11

♫ One of the typical ways to identify an *asymmetrical meter* is noting that the upper number in the *time signature* is a prime number greater than 3. Prime numbers are those that can only be equally divided by 1 and themselves. For our musical purposes here, let's say the typical *asymmetrical meter* will not have an upper number equally divisible by 2 or 3. This principle can be applied to the three examples we've seen already. The top numbers for each have been prime numbers (5, 7, 11) and not divisible by 2 or 3.

The operative word in the paragraph above is "typical." Sometimes, the typical approach is not the one the composer utilizes (they're tricky that way). On occasion, what appears to be a traditional *simple* or *compound meter* gets divided unevenly to create asymmetrical groupings. Here's an example:



You would think that 8 would be divided equally into 4 groups of 2… a nice *simple meter*. What we see here is an unbalanced, asymmetrical division of the *measure*. The first two *beats* are *compound* (3/8) and the third is *simple* (2/8): 3 + 3 + 2 = 8

The same could be done with what we'd consider a typical *compound meter*. When we see 9/8, we would probably assume it was *compound*: {3($\frac{1}{8}$) + 3($\frac{1}{8}$) + 3($\frac{1}{8}$)}. But… The *eighths* in the *measure* could be grouped differently to create an *asymmetrical meter*: 2 + 2 + 2 + 3 = 9

If you're conducting, or even just tapping your foot, *asymmetrical meter* will seem a bit unstable, like that restaurant chair. It may give you the impression of taking a walk on uneven ground where your pace is altered because your foot hits the ground a bit earlier or later than you expected. However you'd like to describe what it feels like, *asymmetrical meter* can be challenging, but fun to listen to and perform.

Section 16

Diatonic Harmony Triad Construction

If you remember our construction metaphor from a couple of sections back, you'll realize in this section that it is doubly appropriate. We are actually going to build vertical *harmonic structures*. When we say "vertical" we're referring to *pitches* that sound simultaneously - *notes* that are arranged on top of each other on the *staff*. The term *"chord"* is commonly used to identify these constructions.

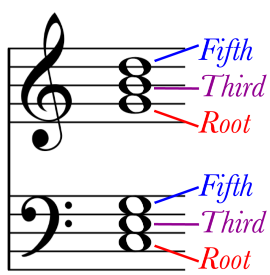
♫ For our purposes, at this point, a *chord* can be any vertical *harmonic structure* that includes three or more *notes*. Some folks will make the case that a construction of only two *notes* can be referred to as a *chord*. As far as this study is concerned, if we only have two *notes*, we'll simply refer to them as a particular *interval*. I must add, though, that in certain *harmonic* contexts, two *notes* can actually imply and substitute for a specific *chord*. (We'll save that discussion for another day.)

In this section we are going to deal with a very specific type of *chord* construction. We will be examining *diatonic triads*. ♫ *"Diatonic"* means that they will function within a traditional *diatonic scale* and *key center*. *"Triad"* means that they will be constructed with three distinct *pitches* in *intervals* of *thirds*.

Our understanding of *intervals* will definitely be important here, serving as a foundation for the structures we're getting ready to build. At this point, our set of building blocks only includes three *pitches* and the *intervals* of *thirds* and *fifths*.

♫ A *triad* has three distinct *pitches*. The foundation of the *triad* is one *pitch*, referred to as *the root*. There will be a *third* above *the root*, referred to as *the third* (imagine that!), and another *third* above *the third*. This top *note* will be called *the fifth* because it is the *interval* of a *fifth* above *the root*. Is that all as clear as mud?

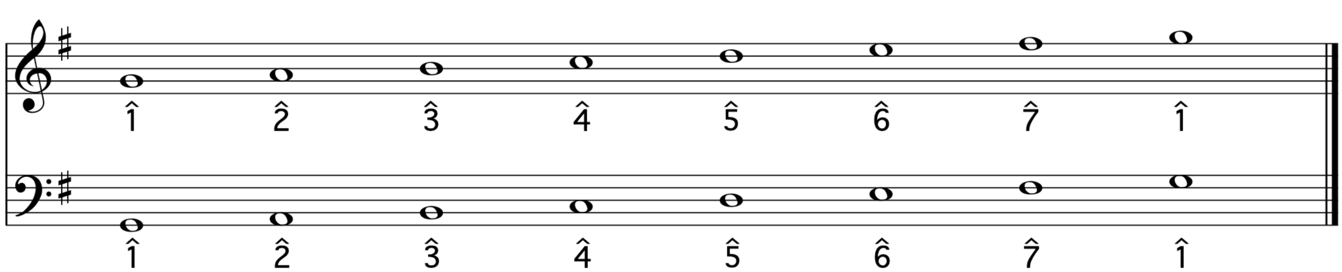
Sometimes words need pictures before they come into focus. Here's what typical *triads* will look like (except for the labels).



As you can see, they have a very distinctive look. On the *staff*, the *pitches* will be on consecutive *lines* or *spaces*.

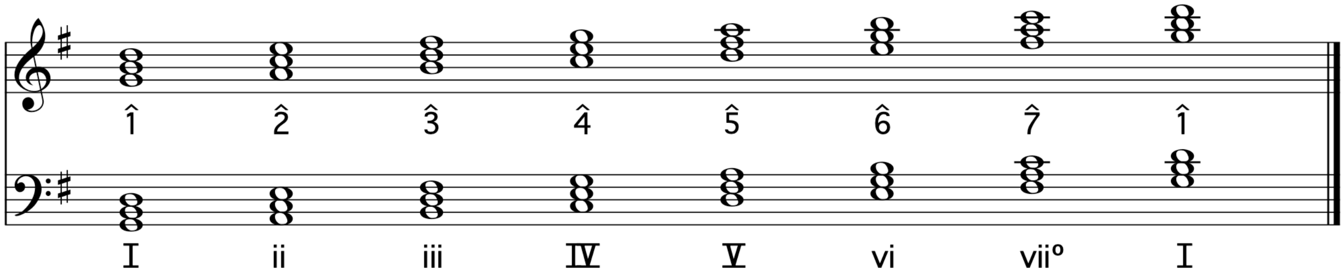
We mentioned above that these will *function* within a *diatonic scale/key*. So, let's consider how *triads* and *scales* interconnect.

First, we'll establish a *diatonic scale*. Let's use *G major*.



Remember our *scale degrees* with the circumflex on top? Our *diatonic triads* will also be numbered…

…but with Roman numerals that coincide with the *scale degrees*. You'll notice that some are upper case and some are lower. Those designations have to do with the quality of the *chord*. We'll discuss those in detail in the next section.



Let's look at the I chord. It's built on the first *scale degree* ($\hat{1}$). The *third* of the *chord* is the third *scale degree* ($\hat{3}$). The *fifth* is the fifth *scale degree* ($\hat{5}$). That seems really simple and straightforward. It becomes a bit trickier with *chords* built on other *scale degrees*.

Let's look at the V *chord*.

fifth = $\hat{2}$
third = $\hat{7}$
root = $\hat{5}$

That doesn't seem as simple. So, is there a way to figure what *scale degrees* are included in a *diatonic triad*? Of course… we just need a bit of math magic.

Thinking back, when we were working on *intervals*, we would figure each *interval* as a set of two *pitches*. It will work the same way with *triads*. They will just have three *pitches* instead of two. Let's look at three simple *triad* sets. This should help us formulate an equation. Remember, we are working with *diatonic scale degrees* and not the *chromatic pitch set* (CPS), so modulo 7 is the system of numbering we'll use.

If we look at the triads on the staff above we'll see that the set of *pitches* for the I *chord* will be $\{\hat{1},\hat{3},\hat{5}\}$ (mod 7). The set for the ii *chord* will be $\{\hat{2},\hat{4},\hat{6}\}$ (mod 7). Likewise… iii = $\{\hat{3},\hat{5},\hat{7}\}$ (mod 7) We should be seeing a pattern, right? ♫ Each *pitch* of the *triad* is two *scale degrees* above the previous.

♫ For our equation, we will us p as the variable. It will represent the *scale degree* (*pitch*) of the *root* of the *triad*. Our equation will look like this:

$$\textit{diatonic triad} = \{\underbrace{p}_{\text{root}}, \underbrace{(p+2)}_{\text{third}}, \underbrace{(p+4)}_{\text{fifth}}\} \pmod 7$$

If we were to build a *diatonic triad* on the $\hat{6}$, then p would equal 6. The *triad* would be labeled as vi.

$$\text{vi} = \{6, (6+2), (6+4)\} \pmod 7$$
$$\text{vi} = \{\hat{6}, \hat{1}, \hat{3}\} \pmod 7$$

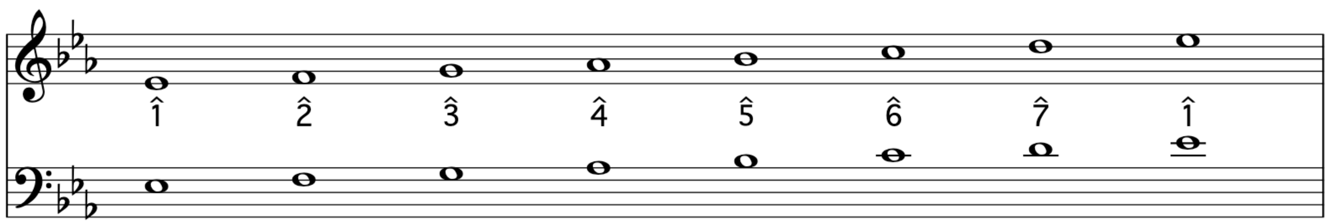
Remember, in modulo 7, $8 \equiv 1$ and $10 \equiv 3$. 8 is congruent with 1 and 10 is congruent with 3. This equation will work for any *diatonic triad* in any *scale*, but… if we think about it… every individual *diatonic triad* will have the same set of *scale degrees* no matter what the *scale* is. Why is that?

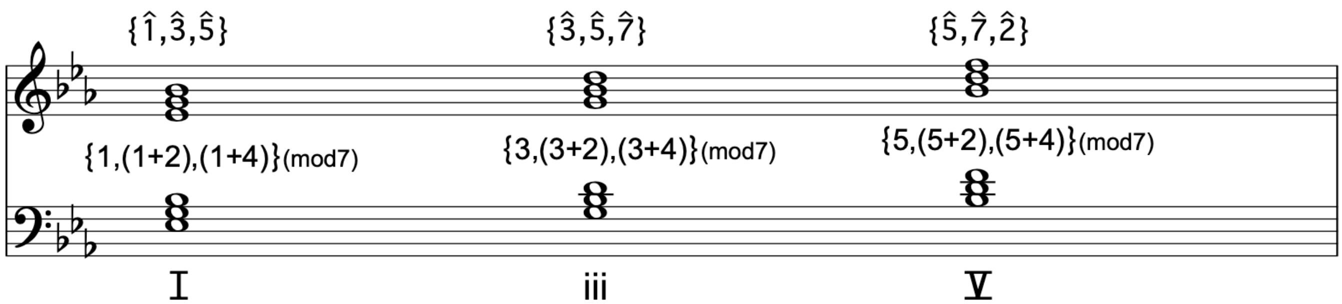
Since the *scale degree* numbers are relative to the specific *scale* (the *tonic pitch* is always $\hat{1}$) and not fixed to a predetermined *pitch* (like in the CPS), every *diatonic triad* is relative to the *tonic pitch* as well.

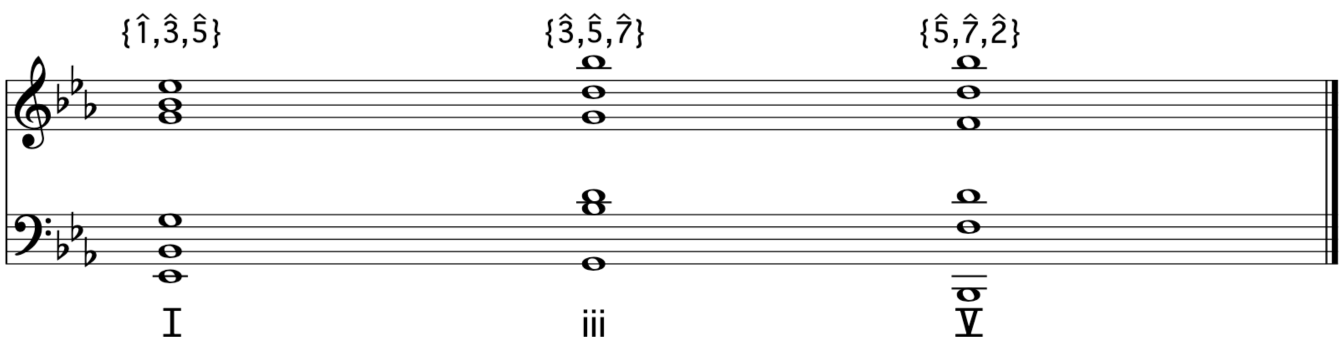
Think about this… I will always be $\{\hat{1},\hat{3},\hat{5}\}$(mod 7). ii will always be $\{\hat{2},\hat{4},\hat{6}\}$(mod 7). iii will always be $\{\hat{3},\hat{5},\hat{7}\}$(mod 7), and so on. It just makes sense. Right?

Let's touch on one more tidbit before we move into the next phase. Let's refer to this bit as *octave displacement*. Now that you have the skills to build *triads*, you should be aware that they don't always show up as a tight group of *thirds*. Sometimes, the *chord members* might be displaced by an *octave*… or two. That *chord member* is still the same *pitch*, it will just be sounding in a different *octave*. There's no need to panic, it's just a matter of knowing what the *chord members* are supposed to be and locating them in a different space.

Here are some examples in the key of E♭ *major*.







Notice in the open spacing above that all the *chord members* are there, they're just spread out. An E♭ is still an E♭ no matter what *octave* it's in. The same is true for any *pitch*. All of these *chords* are still *triads*. It's just a bit harder to determine the *chord members* when they are displaced and not in a close vertical arrangement. File this away. It will be a conversation we'll come back to later on.

Section 17

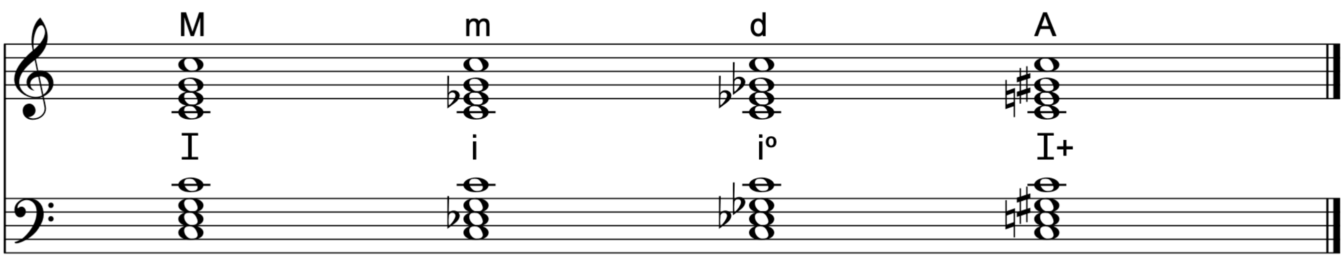
Diatonic Harmony
Chord Quality

In the previous section we saw both upper and lower case Roman numerals used to label our *diatonic triads*. It was mentioned that the different versions of the numerals were associated with the different qualities of the *triads*.

♫ Every *triad* has a quality based on the *interval* relationships within that *triad*. Those relationships are determined by the distance the *third* and the *fifth* are above the *root*. Here's a graphic table that should put this concept into a concise format.

Quality of the triad	Roman numeral type	Distance from root up to third	Distance from root up to fifth
Major	I	M3	P5
Minor	i	m3	P5
Diminished	i°	m3	d5
Augmented	I+	M3	A5

Here's how those would look on the staff if C was the root of the *triad*.



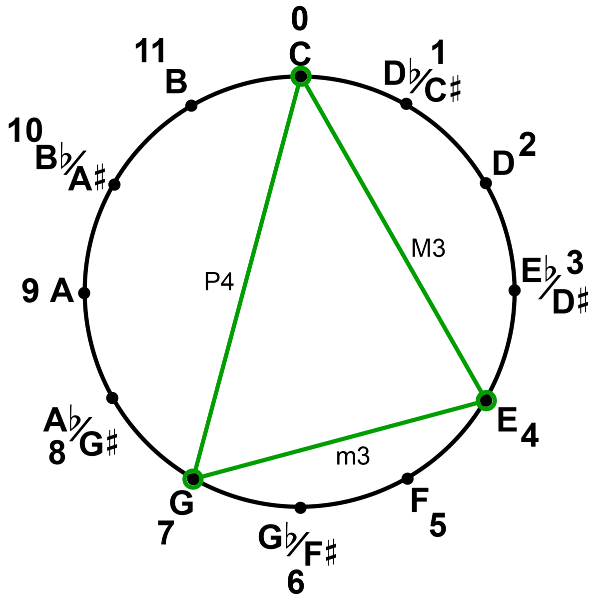
Of course, each *pitch* has relationships with the other *triad* members and not just with the *root*. We should also consider the distance between the *third* and the *fifth* as well as the distance from the *fifth* up to the *root*. How about another table to examine the other relationships?

Quality of the triad	Roman numeral type	Distance from root up to third	Distance from third up to fifth	Distance from fifth up to root
Major	I	M3	m3	P4
Minor	i	m3	M3	P4
Diminished	i°	m3	m3	A4
Augmented	I+	M3	M3	d4

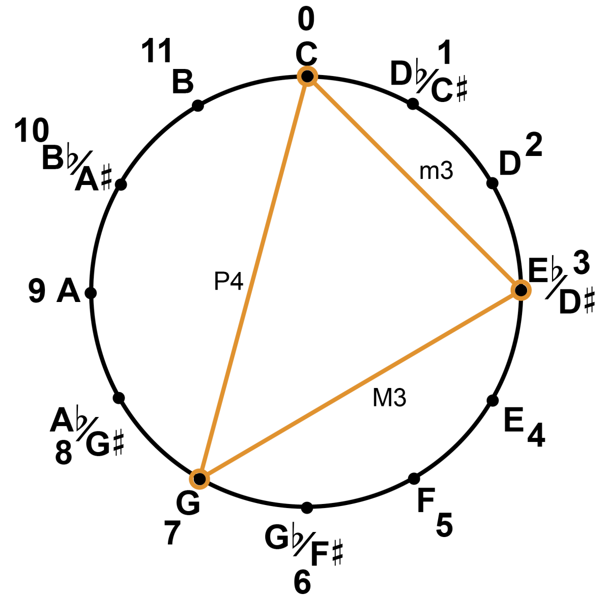
Looking at these relationships on our CPS will give us a type of geometric perspective on how *triads* are constructed.

Here are the four *triads* that we see on our *staff* (previous page) graphed on the CPS.

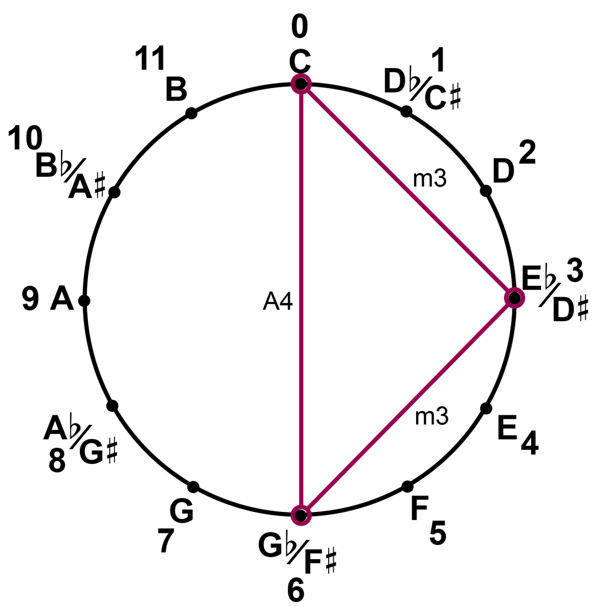
C MAJOR



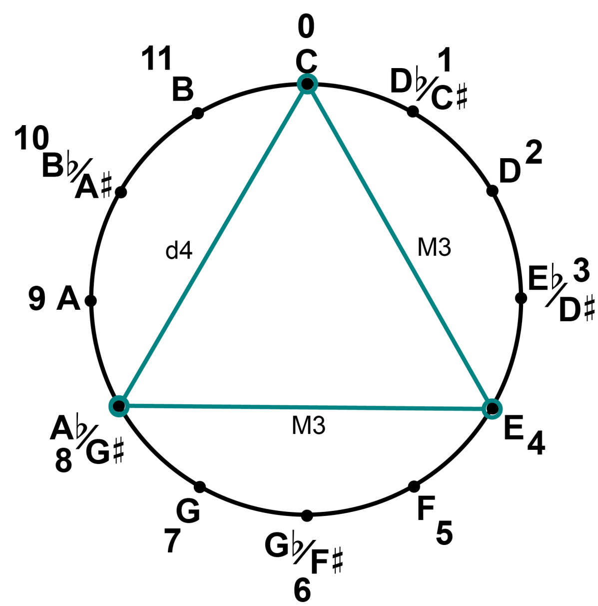
C MINOR



C DIMINISHED



C AUGMENTED



If you are interested in considering other geometric connections between *triads*, check out The Geometry of Triads page.

♫ Remember, the distance from C up to the *fifth* of the *triad* would be clockwise on the CPS. This should make us consider the composite *interval* we get by adding two *intervals* together. You'd think that adding two 3rds together would give us a 6th, but it gives us a 5th instead. The reason for that is, the two *intervals* share a common *note* between them - they overlap. If you add a M3 (C↑E) and a m3 (E↑G) you'll get a P5. Make note of the common E between the two *thirds*. Also, if you add a m3 (C↑E♭) and a M3 (E♭↑G) you'll get a P5. This is the same concept as: 4+3 = 7 or 3+4 = 7. This should be easily apparent on the CPS.

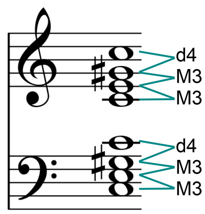
Let's also note, before we move on, that: m3 + m3 = d5 and M3 + M3 = A5.

The *augmented triad* has some interesting tricks up its sleeve. Why don't we take a bit of time and explore those secrets.

Looking at the CPS graph of the *augmented triad* on the previous page you'll see that the triangle it creates equally divides the 12 *pitches* of the *octave* from C to C. If we divide the 12 *pitches* of the *octave* by the 3 *pitches* of an *augmented triad* we get 4. This is the number of *half-steps* between each *pitch* of the *triad*. Remembering our CPS *intervals*, we know that 4 *half-steps* equals a M3.

So, in our example we have:

G♯↑C = d4
E↑G♯ = M3
C↑E = M3



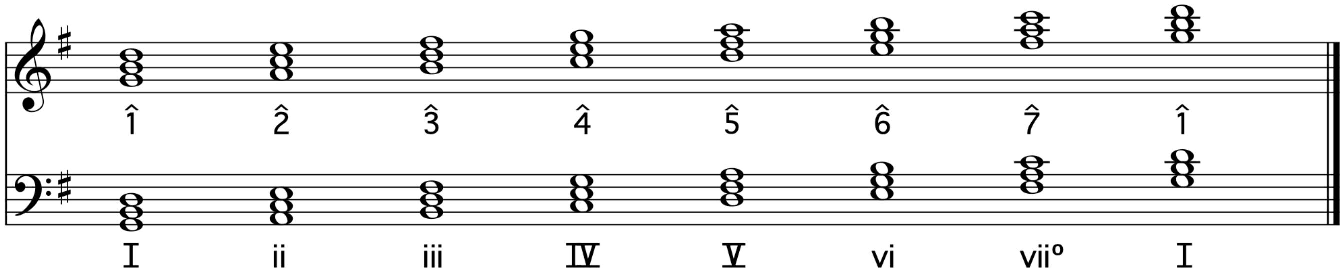
Wait! One of those is not a M3, it's a d4. G♯ to C is not a M3. The secret here is that the *enharmonic* equivalent of G♯ is A♭, and that A♭↑C = M3. So, the math and the music can agree.

♫ The other secret in this is… Any *augmented triad*, spelled *enharmonically*, can be three different *augmented triads*. Each *pitch* of an *augmented triad* can become the *root*. Using our C^+ (C *augmented triad*) as the starting point, we can create the following:

C^+ = {C,E,G♯}
E^+ = {E,G♯,B♯} B♯ respelled *enharmonically* is C
G♯$^+$ = {G♯,B♯,D𝄪} D𝄪 is D *double sharp*, raising D 2 *half-steps enharmonically* to E
This whole triad could be respelled as {A♭,C,E}

Now that we have examined some geometric examples of *chord qualities* and explored some of the secrets of the *augmented triad*, it's time to move on and consider the *triads* in their *diatonic* context.

Let's bring back the notation example from a few pages back. It is a G *major scale* with the *diatonic triads* built on each *scale degree*.



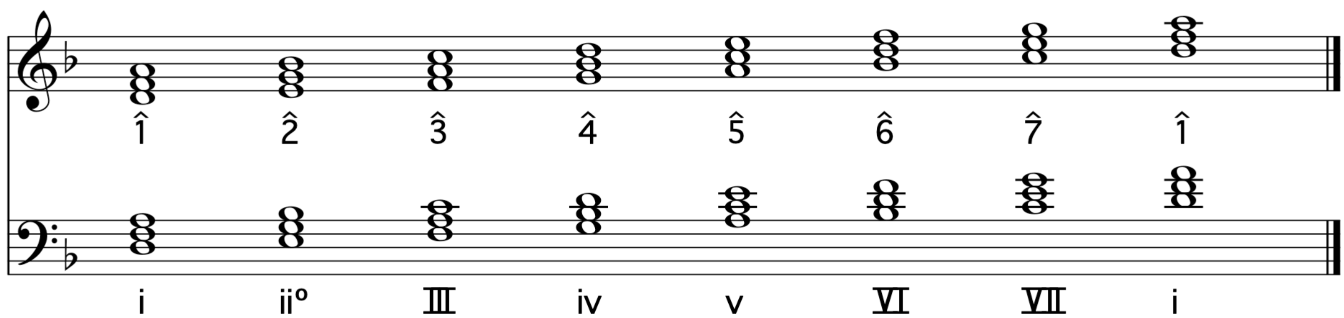
Now that we're familiar with the Roman numeral designations, we can easily determine the *chord quality* of each *triad*, based on its label. ♫ We should make a BIG NOTE right here that says, "the *diatonic chord qualities* will be exactly the same for every *major scale*." Let's sort these *chords* into three groups to consider what we have. We might as well throw in another table to give us some graphic representation.

Chord Qualities in All Major Scales

Chord Quality	Chord Number
Major	I , IV , V
Minor	ii, iii, vi
Diminished	vii °
Augmented	-

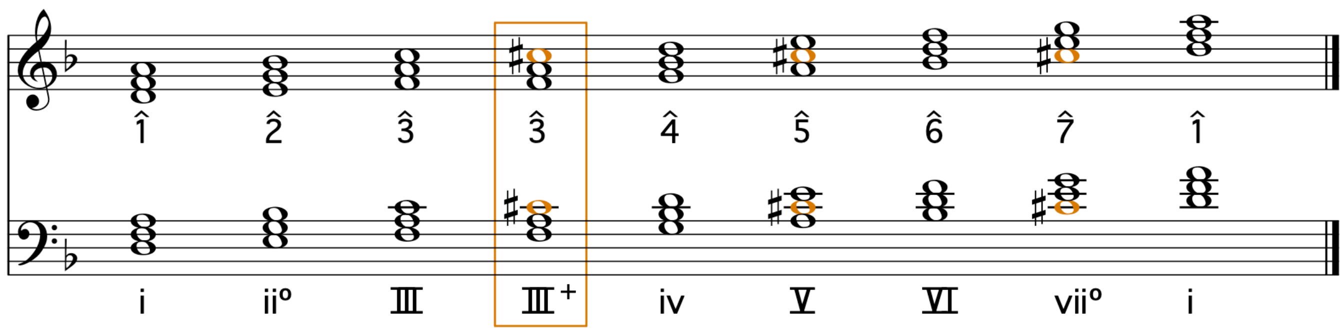
The *major keys* are all straightforward and consistent, but, if you'll remember, we have three different versions of the *minor scale*. The possible *chord* options in a *minor key* will give us a bit more to consider.

D pure/natural minor



The *pure/natural minor*, like the *major*, is fixed and stable. At this point in our study, there are no *altered pitches* or *chords*. Unfortunately, for our analysis purposes on down the road, the *natural minor* is generally not the one we'll see most often. The *harmonic minor* is probably the most common.

D harmonic minor



Note each of the raised *leading tones* (in orange) in our example immediately above. ♫ This altered $\hat{7}$ allows the *harmony* to have a *major* V. That's why this version is referred to as the "harmonic." The *scale* is altered to accommodate the *harmonic* structure. The *major* V, for all intents and purposes (from

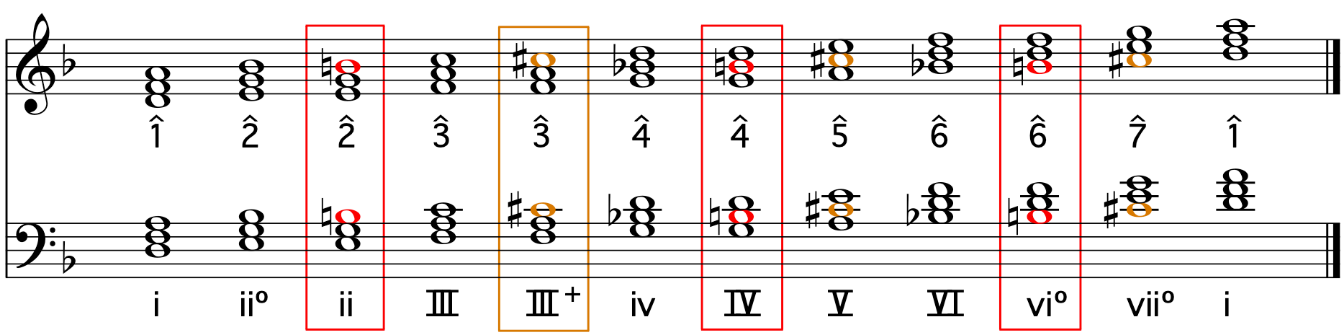
16th-century English law), is the "strongest" *chord* in the *diatonic* system of composition. It pushes the *harmony* toward the *tonal center* like no other *chord* in the system. So, the *pure minor scale* is altered in favor of a strong *harmonic function*. This is a topic we'll cover in more detail very soon.

There's one other issue to note in our example of the *harmonic minor scale* - the III+ *chord*. In a *minor key*, the *chord* built on the $\hat{3}$ is typically a *major chord*. ♫ If you'll remember, the III of the *minor scale* is the I of its *relative major key*. Making the III into an *augmented chord*, by including the raised $\hat{7}$, certainly breaks down that relationship. This III+ is included in the example, but isolated in a box to indicate that it is a possibility but not often used.

With the inclusion of the raised $\hat{6}$ and $\hat{7}$, the *ascending melodic minor* certainly has a variety of colorful *chords*. As in the previous example, we've isolated the possible but uncommon *triads* in boxes.

The altered *pitches* in this version of the *minor scale* are more commonly seen in *melodic lines* with less tendency to dictate the *harmony*. The name of the *scale* itself indicates this tendency. Don't forget… the *descending melodic minor* is exactly the same as the *pure minor* with no *altered pitches.*

D melodic minor



Even though it will be a bit more of a challenge, let's add a table to indicate the chord qualities that are possible in the minor scales.

Possible Chord Qualities in the Minor Scales

Chord Quality	In the Pure Minor	In the Harmonic Minor	In the Melodic Minor
Major	III , VI , VII	III , V , VI	III , [IV], V , VI, VII
Minor	i, iv, v	i, iv	i, [ii], iv, v
Diminished	ii°	ii°, vii°	ii°, [vi°], vii°
Augmented	-	[III+]	[III+]

Those *chords* in orange have a raised $\hat{7}$.
Those *chords* in red have a raised $\hat{6}$.
Chords in brackets indicate a possible but not typical option.

Before we wrap up this section, let's make sure we cover the basic math equations for our *chord qualities*. Here's a table to remind us of the *interval relationships* for each, by the numbers.

Chord Quality	Equation (p = pitch of the root)
Major	{p,(p+4),(p+7)} (mod 12)
Minor	{p,(p+3),(p+7)} (mod 12)
Diminished	{p,(p+3),(p+6)} (mod 12)
Augmented	{p,(p+4),(p+8)} (mod 12)

Section 18

Diatonic Harmony Seventh Chords

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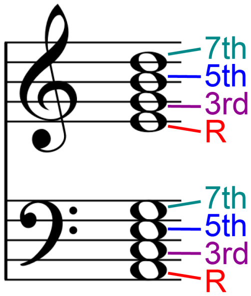
It's time to add another *third* on top of our *triads*. Up to this point, we've had a *root*, a *third*, and a *fifth*. When we add another *third* on top of the *fifth* we'll get a *seventh*. According to Spock, logic dictates that a *third* on top of the *fifth* will be a *seventh* above the *root*. It will look like this (without the labels):



Like our *triads*, each *7th chord* will have a (double) *quality* based on the *intervals* above the *root*. These two *chords*, noted above, are both mm *chords*. The first *quality* in the label will be for the *triad* and the second *quality* will be for the 7th. So, these *chords* have a *minor triad* with a *minor* 7th.

m m

minor triad   minor 7th

As with our *triads*, let's put together a chart to round up and corral the <u>common</u> *diatonic* options.

Quality of the 7th chord	Roman numeral type	Distance from R up to 3rd	Distance from R up to 5th	Distance from R up to 7th
Major-Major	I^{7} (I^{maj7})	M3	P5	M7
Major-Minor *	I^{7}	M3	P5	m7
Minor-Major	i^{7} (i^{maj7})	m3	P5	M7
Minor-Minor	i^{7}	m3	P5	m7
Diminished-Minor **	$\text{i}^{ø7}$	m3	d5	m7
Diminished-Diminished ***	i^{o7}	m3	d5	d7
Augmented-Major	I^{+7}	M3	A5	M7

The Roman numeral chord designations in parentheses are optional
* The *major-minor 7th chord* is often referred to as the *dominant 7th chord*
** The *diminished-minor 7th chord* is referred to as the *half-diminished 7th chord*
*** The *diminished-diminished 7th chord* is referred to as the *fully-diminished 7th chord*
(more on all of these later)

If you'll note, our chart above contains <u>common</u> *diatonic 7th chords*. There are certainly other options for combining *triad* and *7th qualities*, but most of those do not commonly occur in traditional *diatonic scale structures.* You'll certainly find those alternative *7th chords* in jazz harmony. (…but that's another course for another time.)

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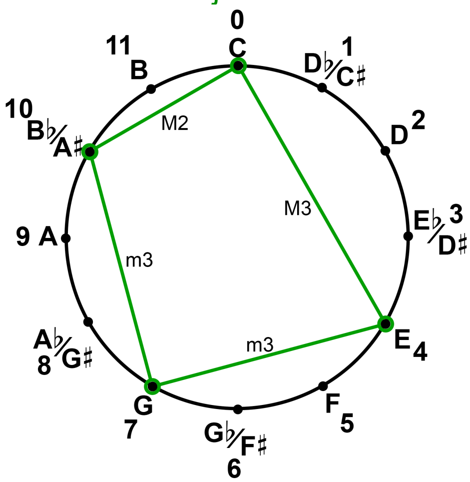
Before we discuss the secrets of the *dominant 7th chord* and the *diminished 7th chords*, let's look at the other *third* relationships of the *chords* included in the chart above. …in yet another chart, perhaps?

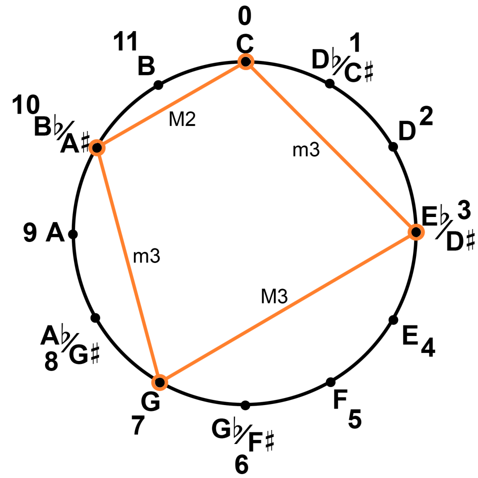
Quality of the 7th chord	Distance from R up to 3rd	Distance from 3rd up to 5th	Distance from 5th up 7th	Distance from 7th up to R
MM	M3	m3	M3	m2
Mm	M3	m3	m3	M2
mM	m3	M3	M3	m2
mm	m3	M3	m3	M2
dm	m3	m3	M3	M2
dd	m3	m3	m3	A2
AM	M3	M3	m3	m2

We're not going to graph every possible *7th chord* on the CPS, but it might help us to look at a few of the more common combinations. (If you're interested in more advanced geometric connections click here.)

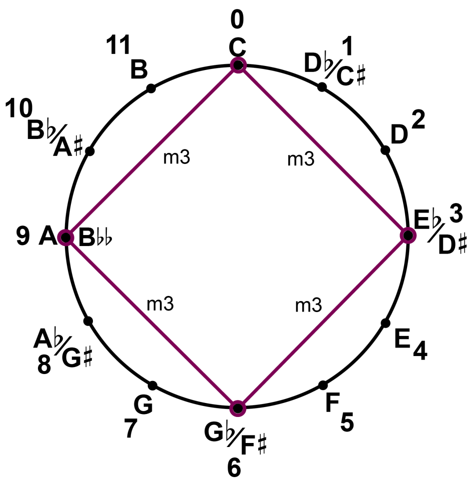
C Major-Minor



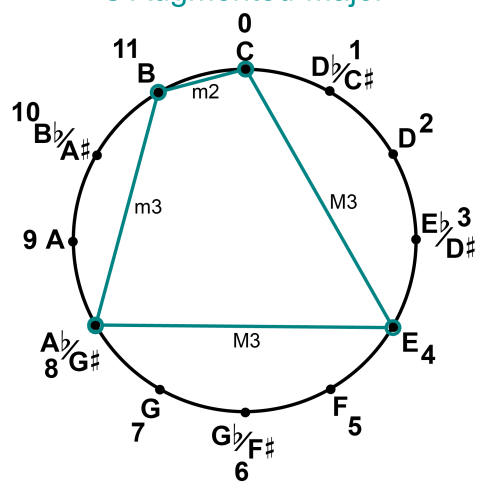
C Minor-Minor



C Diminished-Diminished



C Augmented-Major



Let's rewind a bit and take a look at the Roman numeral options noted in the chart a couple of pages back. Here's what was included in the chart:

Ⅰ^{7} (Ⅰ^{maj7})
i^{7} (i^{maj7})

♫ In a strictly *diatonic* setting, a Ⅰ^{7} will always be a MM *7th chord*. The *major* Ⅰ *chord* will only exist in a *major key*. That *chord* will not exist in any of the *minor keys*. Also, in a *major key*, the 7th of the Ⅰ *chord* will always be *major*. The 7th of the Ⅰ^{7} will be the $\hat{7}$, which is a M7 above the *tonic pitch*, which is the *root* of the Ⅰ. From this perspective, the optional Ⅰ^{maj7} label will never be necessary.

Labeling the i^{7} in a *minor key* can be a bit tricky. Suffice it to say, the actual music will determine the kind of label used. Since the *minor key* has so many variants, there are a few scenarios to consider, but remember… the i *chord* will always be m (*minor*).

- In a passage of music that utilizes the *pure minor* form, a i^{7} will be a mm *chord*.
  The 7th of the i^{7} will be the $\hat{7}$, which is a m7 above the *tonic pitch*, the *root* of i
  In this scenario, the optional i^{maj7} label is incorrect and inappropriate
- In a passage that utilizes the *harmonic minor* form, the i^{7} would probably be a mm chord
  In the *harmonic minor*, the raised $\hat{7}$ is typically used in a *harmonic function* with a major Ⅴ
  The raised $\hat{7}$ will probably not be used with a i chord (the same goes for Ⅲ), but…
  IF the raised $\hat{7}$ is used with the i *chord*, the optional i^{maj7} should probably be used
- When the *melodic minor* version is in play, it may be necessary to utilize the optional ^{maj7} label, dependent on what's actually happening in the music at the time.

Of course, you realize there are six other *chords* in a *diatonic scale* and we've only been considering the Ⅰ. ♫ The bottom line is this… if the 7th of any *diatonic chord* lies within its unaltered *diatonic scale*, the label only needs the ^{7}, nothing else. If, in a *minor key*, the altered *scale degrees* within the music take a *chord's* 7th away from what's found in the *pure minor*, the labeling will need to be adjusted to make it clear how the *chord* is actually being structured.

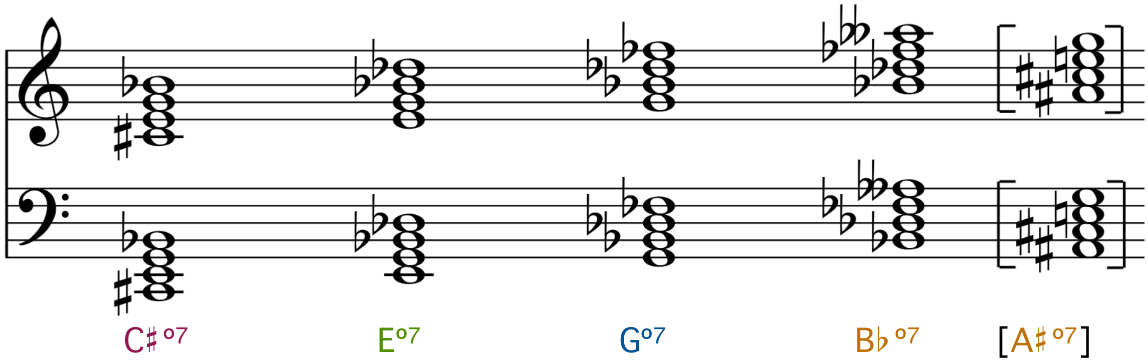
We've come to the point where we dive in to the secrets of the *diminished seventh chords*. To be honest, there's really only one secret, but it definitely holds a bit of magic. First we need to consider the two types of *diminished 7th chords*. If you took note of our first table in this section, you'll remember that a dm *seventh chord* is referred to as a *half-diminished 7th*. The dd *seventh chord* is called *fully-diminished*. It's given that distinction because both qualities, the *triad* and the *7th*, are *diminished*. By that reasoning, we should be quick to figure out why the other is *half-diminished*.

The *fully-diminished 7th chord* is the one that can produce musical magic. If you remember how our *augmented triad* equally divided the *octave* because it is built with all M3s, you should realize that our dd7 does the same thing with m3s (see the CPS on the previous page). ♫ That, in itself, is kind of fun to consider, but the real trick is being able to build four different dd7 *chords* with the *pitches* of one *chord*. Like the *augmented triad*, each *pitch* of the dd7 can be the *root* of a different dd7 *chord*. There will be several *enharmonic* respellings, but the sounding quality of the *chord* remains the same. Let's stack up a dd7 chord with C♯ as the *root* and then shift the *root* through each chord member to see what happens.

Notice in the graph below how the R (*root*) of a *chord* is rotated around to become the 7th of the next *chord*, leaving the 3rd to become the R (*root*) of the next *chord*.

C♯	E	G	B♭			
	E	G	B♭	D♭		
		G	B♭	D♭	F♭	
			A♯	C♯	E	G

Also, take note of the *enharmonic* respellings. Doing this keeps the *third* relationships intact for each successive rotation of the *chord members*. To avoid *double flats* in this situation, the last *chord* is respelled using *sharps*.



You're probably wondering, "what's the point?" It's this. ♫ As we keep traveling down this road, we'll run into situations where there is a need to transition (*modulate*) from one *key center* to others. The dd7 chord typically *functions* as a vii^{o7}, which has a very strong tendency to progress (*resolve*) to I (or i). Respelling the vii^{o7} allows it to *function* in two different *key centers* and serve as a *pivot* point from one to the next. It will begin in one *key*, be respelled, and *resolve* to I (or i) in the new *key*.

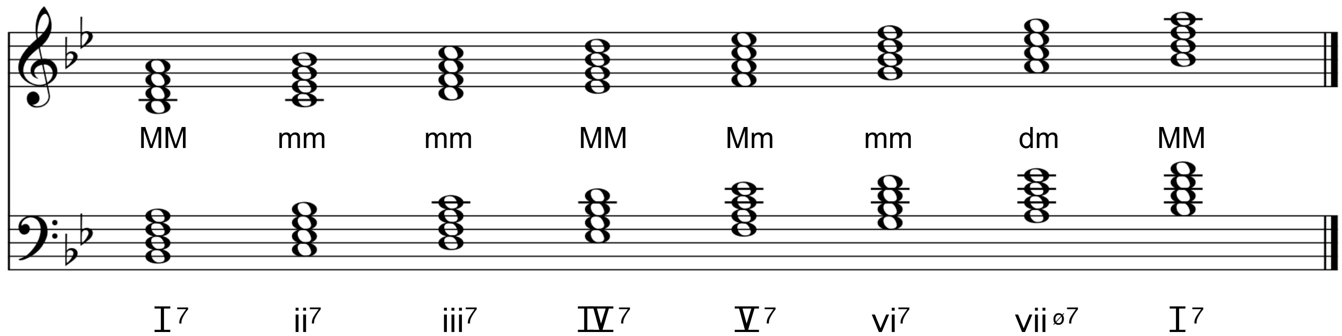
So, we've just gotten a glimpse of the future. File this away and when our travels bring us to encounter these situations, we will be prepared with a bit of magic to see us through.

Before we forget, let's talk about the Mm7. As we noted in the the chart three pages back, this *chord* is generally referred to as the *dominant 7th chord*. This is the *chord quality* of the V^7 — the *7th chord* built on the *dominant scale degree* (remember our discussion of the names of the scale degrees?). This Mm quality holds an extremely prominent role in the functions of traditional *diatonic harmony*. The *dominant 7th chord* contains the *leading tone*, which is, what we call, an *active tone*. In a *diatonic* context, the *leading tone* ($\hat{7}$) actively leads or pushes our sense of resolution toward the *tonic pitch* ($\hat{1}$).

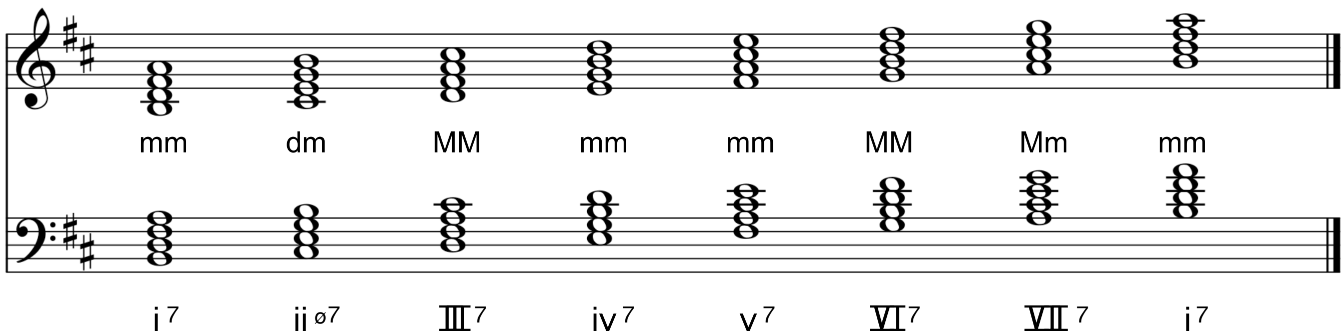
This *chord* also contains a *tritone*, between the 3rd of the *chord* and the 7th of the *chord*. This active *interval*, begs for *resolution.* The 3rd (which is the *leading tone*) wants to *resolve* to $\hat{1}$ and the 7th wants to *resolve* down to the $\hat{3}$. V^7 really wants to resolve to I . This strong tendency makes the *dominant 7th chord* the most active and vital *chord* in *diatonic harmony*. We'll really dive in to all of this when we get to Harmonic Progression.

We should take a quick peek at the typical *diatonic 7th chords* on the *staff*.

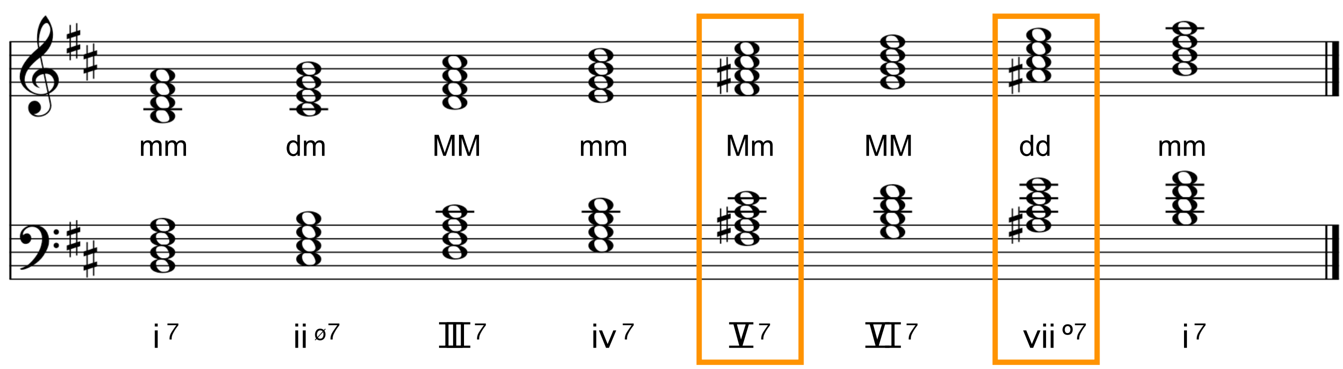
B♭ major



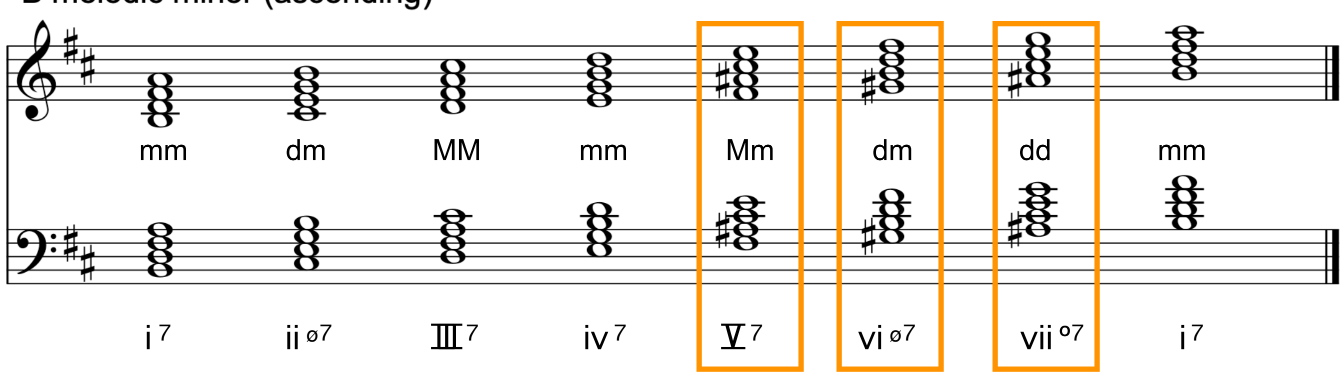
B pure minor



B harmonic minor



B melodic minor (ascending)



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As usual, please note that these are just examples and that these *chord qualities* will be consistent no matter what *key* is being utilized.

Before we hit the finish line on this section we need to consider this last set of *7th chords* for the *melodic minor*. Since the *scale* can contain both the raised $\hat{6}$ and $\hat{7}$, the *harmony* could go in several directions, giving us multiple options for these *7th chords*. What you see above are what we should consider "typical."

But…

If $\hat{6}$ is raised:

- ii ^{7} could be a mm
- iv ^{7} could be Mm (IV ^{7}) (this would make it a *dominant 7th chord* which would have a tendency to lead us to another key)

If $\hat{7}$ is raised:

- i ^{7} could be a mM (i ^{maj7})
- III ^{7} could be a AM (III $^{+7}$)

There it is!

Let's move on.

Section 19

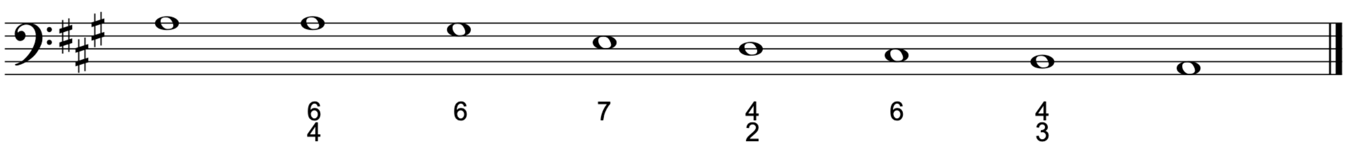
Diatonic Harmony
Inversions and Figured Bass

What if we have a *chord* in which the *root* is not the lowest *note*? What if the 3rd is the lowest *note*… or the 5th? Is it the same *chord*? Does it function the same way? Is there a way to label the different versions?

♫ If a *chord* has one of its members, other than the *root*, as the lowest-sounding *note*, that *chord* is considered to be *inverted*. In most respects, it is the same *chord* with the same Roman numeral designation, but the function of some particular *chords* will change a bit based on the musical context.

To answer our last question… Yes. There is a way to label the different versions. We refer to this system as *figured bass*. It is a kind of musical shorthand that was used in the 17th and 18th centuries. This era, the Baroque and early Classic periods, utilized a practice referred to as *continuo*. This was a type of basic *accompaniment* that included an instrument, such as a cello, playing a *bass line*, and a keyboard instrument, such as a harpsichord, filling in the *harmony*. (Should someone make a joke here about adding a drum set and making it a jazz trio?!)

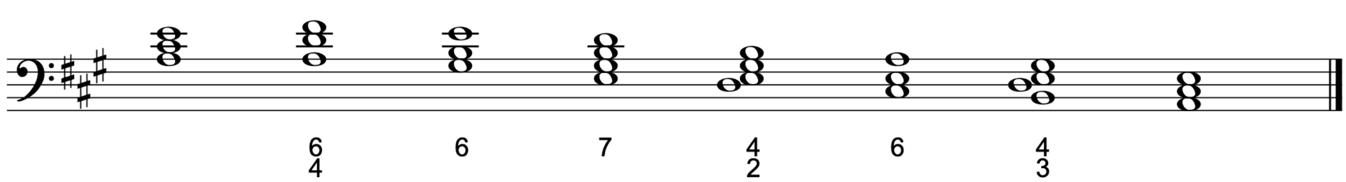
This shorthand musical *notation* included a *bass note/line* on the *staff* with numbers underneath that indicated the *harmony* to be used. It would look something like this:



♫ The numbers you see under the *staff* indicate *intervals* that should be played above the given *bass note*. It is understood that the *notes* without numbers are in *root position* and that there would be a 3rd and a 5th above the *bass note* (the *root* of the *chord*).

♫ We need to take note at this point. When we use the phrase *bass note*, we are referring to the *lowest-sounding pitch*. Take this little scrap of information and file it away. It will come in handy later on.

When someone plays from this kind of *score*, it becomes *realized*. Below is a *realized* version of the *figured bass score* above.



Notice…

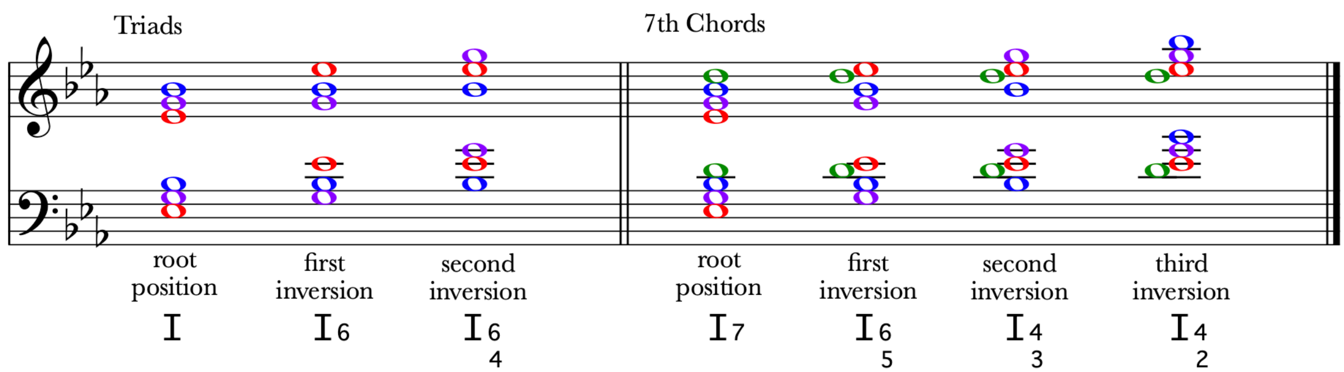
- the first and last *chords* have no numbers. They are in *root position*.
- the third and sixth *chords* only have a 6, but there's a 3rd above the *bass* as well. The 3rd is implied.
- the fourth *chord* is actually in *root position*. The 7 is there to indicate that it is a *7th chord*.
- the fifth and seventh chords are *7th chords* as well. The 6th above the *bass* is implied.

It's time to throw a chart on the page to bring all of this into better focus.

Inversions and Figured Bass

Chord member in the bass	R (triad)	3rd (triad)	5th (triad)	R (7th chord)	3rd (7th chord)	5th (7th chord)	7th (7th chord)
Complete figured bass	5 3	6 3	6 4	7 5 3	6 5 3	6 4 3	6 4 2
Figured bass typically used	(nothing)	6	6 4	7	6 5	4 3	4 2
How to find the root	lowest-sounding note	6th above the bass	4th above the bass	lowest-sounding note	6th above the bass	4th above the bass	2nd above the bass

Why don't we see how these look on the staff!?

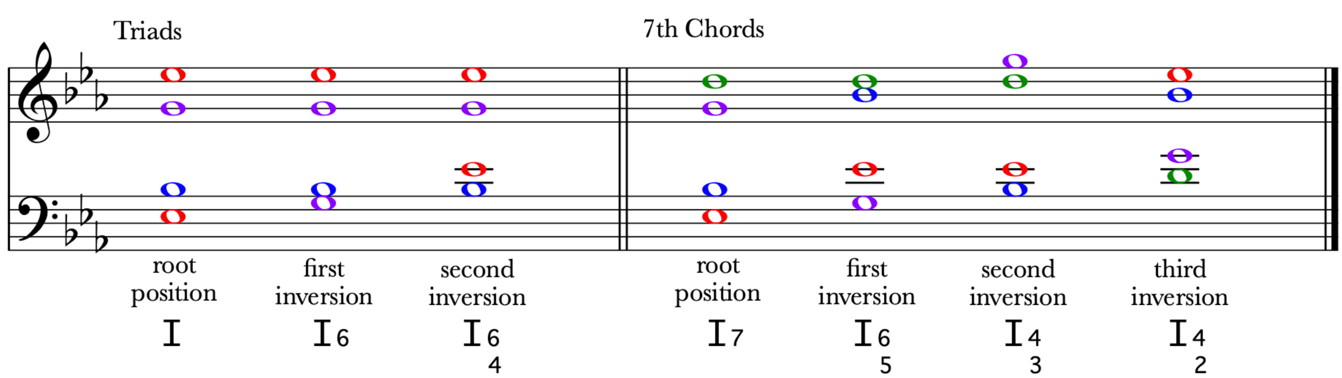


You'll notice in the example immediately above, each type of *inversion* has a designation. If the 3rd is the *lowest-sounding note*, it's considered *first inversion*. …5th, *second inversion*. …7th, *third inversion*.

Also, we've added Roman numerals to the mix. Although we've already discussed the use of these, we haven't really considered where they came from. Historically, composers came first and the theorists followed. Composers would write the music they believed sounded appropriate, according to a system they created or adhered to. The theorists would study, dissect, and analyze the music being written and document what they believed to be the practices being utilized.

Jean-Philippe Rameau, a French composer and theorist of the 18th century, is traditionally credited with the system of labeling *chords* with Roman numerals based on a *chord's root*. What we use today is a combination of Roman numerals, indicating what *root note* the *chord* is built from, and the *figured bass* numbers, indicating the *chord's inversion*.

Here's a quick note: A chord can be opened up with its members spread out over multiple octaves.



Notice in the previous example:

The I should have a 3rd and a 5th above the *bass*. The 5th is there, but the 3rd is an 8va higher.

The I^{6}_{5} has the 6th directly above the *bass*, but the 5th is an 8va higher.

These *octave* displacements don't present any problems with the analysis, except… we have to be able to recognized the *chord members* in whatever *octave* they appear. The given example makes it simple by color-coding the *chord* members. Unfortunately, when we analyze a piece of music, that music won't be that kind of colorful.

Since Rameau's writings on music theory, we have accepted the fact that a *chord*, even if *inverted*, is still the same fundamental *chord*. Our use of Roman numerals verifies this concept, with *figured bass* as added footnotes. This answers another one of the questions posed at the beginning of this section.

One question remains from our opening paragraph. Do *inverted chords* function the same as they do in *root position*? The (less than definitive) answer is… Some do. Some don't.

There is a lot to consider when dealing with *chord function*. Probably, the most significant consideration is the context in which the *chord* is found. Where does it fall in the *key*? What comes before it? What comes after it?

There are two other considerations, in terms of *chord function.* What *inversion* is it in? Does it contain *active tones* (a topic for another day)?

There's one *chord* that can definitely take on other specific functions. The I in *second inversion* (I^{6}_{4}) can be considered a *pre-dominant chord*. This simply means that it can, and often does, precede the *dominant* (V). It can also be used as a *substitute dominant*, taking the spot in a *chord progression* where the *dominant* would usually reside.

Grappling with questions about *chord function* is certainly prompting us to move on to the next section and look for additional clarity. Let's do it!

Section 20

Harmonic Progression
Strong Root Movement

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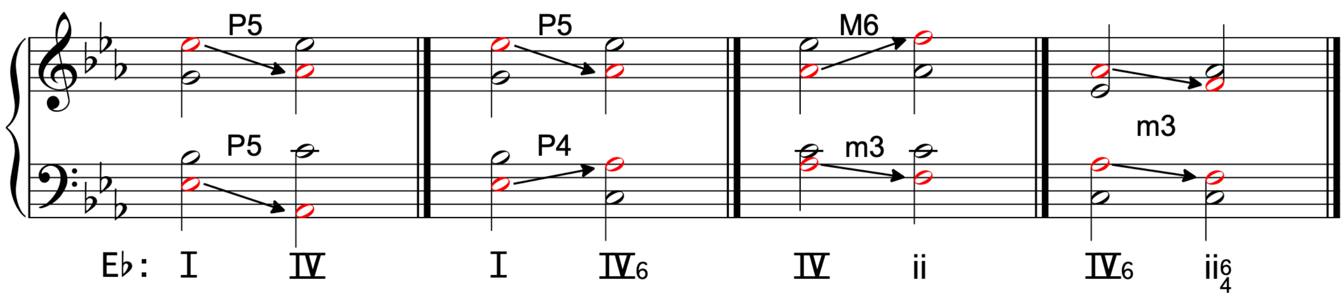
What is *harmonic progression*? It's simply the process of going from one *harmony* to another… from one *chord* to the next. In this process there are two basic types of *movement*, strong and weak. These are not synonymous with "good" and "bad" or "pleasing" and "displeasing." In *tonal* music, strong is generally the norm, but weak is also used in certain instances.

When referring to *harmonic progressions*, Roman numerals for *diatonic chords* are generally used (I, IV, ii, V^7, vi, etc.). ♫ The distance between two *chords* is measured from the *root* of one *chord* to the *root* of the next *chord*, regardless of the *inversions* of the *chords*. ♫ The *note* in the *bass voice* (lowest-sounding pitch) does not determine the distance between the *roots* of the *chords*.

For example:

The distance between I and IV is down a 5th (up a 4th).
The distance between I and IV$_6$ is down a 5th (up a 4th).

The distance between IV and ii is down a 3rd (up a 6th).
The distance between IV$_6$ and ii$^6_4$ is down a 3rd (up a 6th).



Once again, the *chord* members have been color-coded to make it quicker and easier to see the *root movements* of these *chords*. Let's take a short detour and talk about recognizing *chords* when they are not color-coded, in *root position*, or in close position. Here's a quick, two-step process:

1. Find all the pitches included (ignore any doubled pitches)
2. Arrange them in 3rds

Look at two of the *chords* in the example above, the IV$_6$ and the ii$^6_4$.

The *pitches* included in the IV$_6$, from the bottom up, are: C, A♭, E♭
Arranged in 3rds we would have: A♭, C, E♭
A♭ would be the *root*. This is an A♭ *major triad*.

The *pitches* included in the ii$^6_4$, from the bottom up, are: C, F, A♭
Arranged in 3rds we would have: F, A♭, C
F would be the *root*. This is an F *minor triad*.

Many rules and regulations have been formulated over the years to govern what is considered appropriate or *strong harmonic movement*. These rules can be summed up, however, in the simple chart at the top of the next page.

Reminder: These movements are from the *root* of one *chord* to the *root* of the next, regardless of *inversion*.

♫ STRONG ROOT MOVEMENT ♫

↓ 5th	↓ 3rd	↑ 2nd
down a 5th (up a 4th)	down a 3rd (up a 6th)	up a 2nd (down a 7th)
I ⟶ Any I can go to any chord	Two Exceptions ↔	IV ⟶ I IV can go to I

Here are examples of *strong root movement progressions*:

Down a 5th
Examples: I → IV   ii → V   iii → vi

Down a 3rd
Examples: I → vi   vi → IV   IV → ii

Up a 2nd
Examples: I → ii   iii → IV   IV → V

From I to any other *chord*

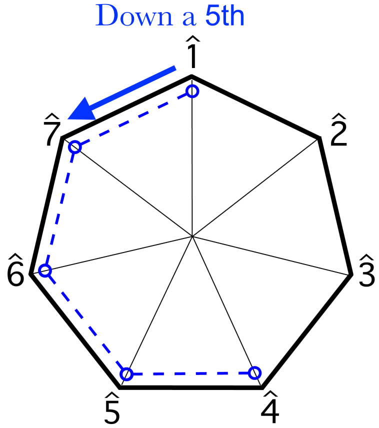
From IV to I

Strong root movement progressions will be what we focus on as we consider *tonal* music from the Common Practice Period (roughly 1650 to the early 20th century). But… as mentioned, *weak root movement progressions* are used in certain instances (at the composer's discretion). For clarity, we should note that these are the exact opposites of *strong movements*: Up a 5th   Up a 3rd   Down a 2nd

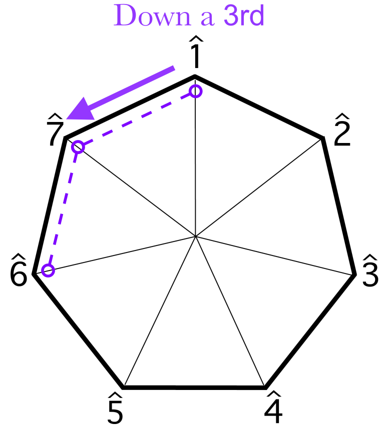
What if we add a little math to this? Maybe it will be just the crutch we need to help us move along!?

Since we are dealing with *diatonic harmony* we are working in Modulo 7. Remember the mod 7 "clock" from Section 1? Here it is again to help us visualize our *harmonic root movements*. Remember… upward movement goes clockwise and downward goes counter-clockwise.

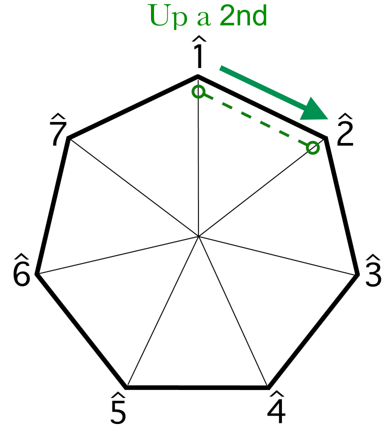
Down a 5th



Down a 3rd



Up a 2nd



These examples all have the *tonic pitch* (the *root* of the I *chord*) as the starting point. As always, these calculations can begin from any *pitch* that serves as the *root* of a *diatonic chord*.

For some quick equations, let r be the variable for the *pitch* that serves as the *root* of the *chord*.

↓5th = (r - 4) (mod 7)
↓3rd = (r - 2) (mod 7)
↑2nd = (r + 1) (mod 7)

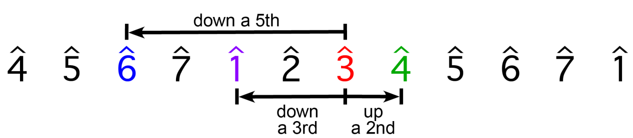
Let's say we have a iii *chord*. What are the *strong root movements* from that *chord*?

If r = $\hat{3}$ then ↓5th = (3 - 4) (mod 7)
↓5th from $\hat{3}$ = $\hat{6}$

If r = $\hat{3}$ then ↓3rd = (3 - 2) (mod 7)
↓3rd from $\hat{3}$ = $\hat{1}$

If r = $\hat{3}$ then ↑2nd = (3 + 1) (mod 7)
↑2nd from $\hat{3}$ = $\hat{4}$

Here's a different graphic form that might help us visualize these figures (note the correlation of colors from the equations above).



(To see an animation of all the options, click here.)

Before we put this to bed, let's look at a few actual *harmonic progressions* to see how they fit together.

Here's one that only uses ↓5th *movements*.

I IV^6_4 $\mathrm{vii}^{\circ}{}_6$ iii vi^6_4 ii_6 V^4_2 I_6

Just as a note… *progressions* can utilize ↓5th *movements* exclusively and be effective. Using ↓3rd or ↑2nd *movements* exclusively is really not very effective. It's better to mix these with the other options. Here are some examples.

↓3rd ↓3rd ↓3rd ↓5th ↓5th
I vi IV_6 ii^6_4 V^6_5 I

↑2nd ↓5th ↑2nd ↓3rd ↑2nd ↓5th
I ii^4_2 V^6_5 vi_6 IV^6_4 V^4_3 I

Section 21

Harmonic Progression Cadences

Did it occur to you that we didn't talk about the IV to I *progression* mentioned in the *Strong Root Movement* chart? The reason is, we were saving it for this section, when we talk about *cadences*.

♫ In traditional *tonal* music, the linear movement is often divided into *phrases*. These are typically a few *measures* in length and end with some sort of pause or, as I like to call it, a point of repose. The most traditional, balanced arrangement is two *phrases*, each usually four or eight *measures* in length. The first *phrase* is referred to as the *antecedent phrase* and the second as the *consequent phrase*. Together, they form what is called a *period*. These two *phrases* (the *period*) are often thought of like a combination of question and answer.

Here's an example of a well-balanced musical *period*:

If you are utilizing the e-book format, you should be able to click on the icon at the end of the example and listen to this music. If you do, you'll be able to hear how each *phrase* ends with a pause, a point of repose. As you would expect, these two *cadences* have labels.

♫ The *cadence* at the end of the first (*antecedent*) *phrase* is what we call a *Half Cadence*. This type of cadence ends with some sort of V chord. It usually occurs halfway through the *period*, at the end of the *antecedent phrase*.

♫ The *cadence* at the end of the *consequent phrase* is called an *authentic cadence*. It is made up of a V to I *progression*. This type of *progression* usually occurs as a final *cadence*, at the end of a *period* or the end of the whole piece.

Of course, there are more than just two. On the following page you'll see an outline of the various types of *cadences*, with an explanation for each.

Authentic Cadence

V or vii° going to I

Perfect Authentic Cadence (PAC)

V going to I

Both V and I are in *root position*

The *tonic pitch* will be in the uppermost *voice* of the I *chord*

Imperfect Authentic Cadence (IAC)

V or vii° going to I

Any *inversion* of either *chord*

Any *pitch* of the I *chord* in the uppermost *voice*

Half Cadence *(HC)*

Any *progression* ending with a V *chord*

Usually seen at the end of the *antecedent phrase*

Plagal Cadence *(PC)*

IV to I

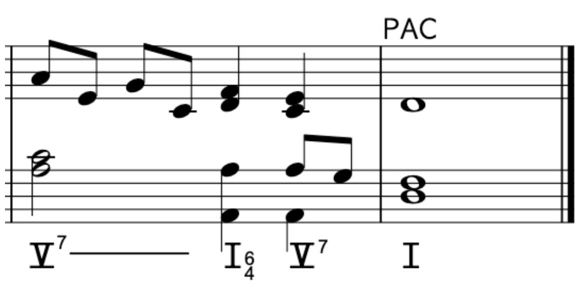
Sometimes referred to as the "amen" *cadence*

Deceptive Cadence *(DC)*

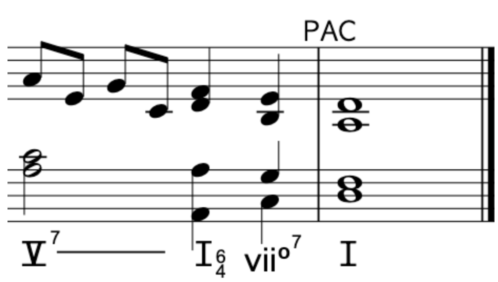
Usually V going to vi

Let's take the musical example from the previous page and re-compose it to show each of the cadences noted above. Since the music already gives us a good example of a *Half Cadence (HC)*, we'll not redo that one. Also, we'll just look at the last two measures of the *consequent phrase.*

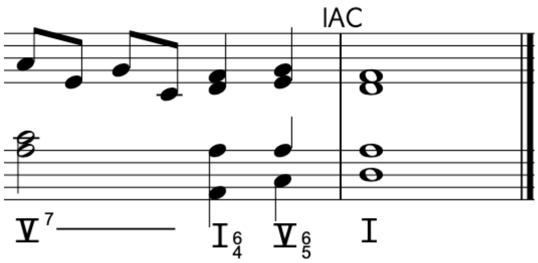
Our musical example already ends with a *Perfect Authentic Cadence…* V to I , both in *root position* with the *tonic pitch* as the highest-sounding *note*. So, there's no need for re-composition. Please note the use of the label *(PAC)*.

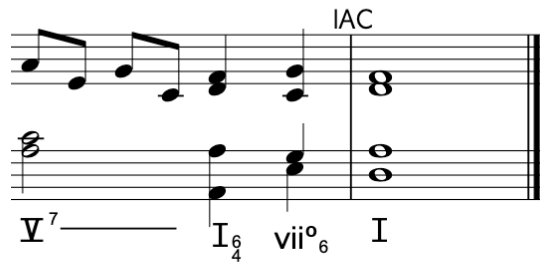


Let's change the V to a vii°. Please note that it is still a *PAC*.



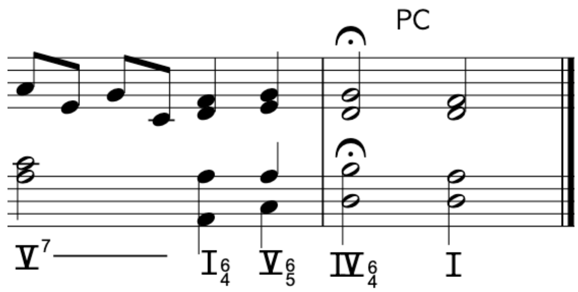
Here are a couple of examples of *Imperfect Authentic Cadences (IAC)*.



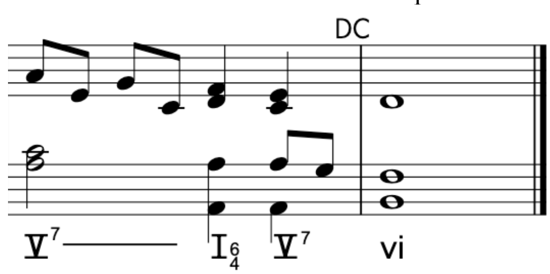


I hope you noticed that both the V and vii° were *inverted* and the I had the *3rd* as the top *note*. One or both of those will make these cadences *IAC*.

The *Plagal cadence*, thought of as the "amen" *cadence*, is often used as a way of extending the phrase and delaying the final chord just a bit. I assume you realize the cadence is the IV to I movement and **not** the V to IV in this example.

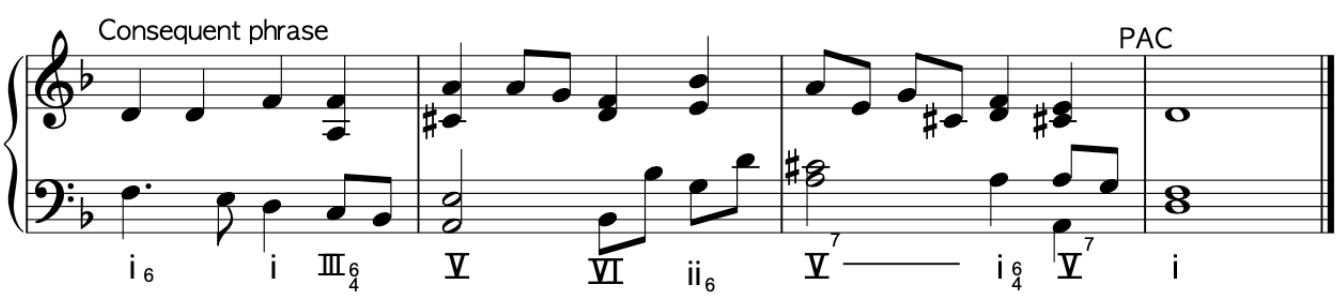


The last *cadence* in our set gets its name from the fact that it takes the *harmony* to an unanticipated resting place. The *Deceptive Cadence* takes the V to vi instead of the expected I .



Just a note: The *Deceptive Cadence* is not usually the final cadence of a piece of music. It is often used to extend the phrase, add another phrase, or to lead the music off in a different direction.

Of course, each of these *cadences* is perfectly suited for *minor keys* as well as the *major*. The example below has been redone in *D minor*.



Section 22

Harmonic Progression Voice-Leading

It's time to turn our eyes away from the vertical and gaze upon the horizontal. We're going to take a little sideline tour away from *harmony* and spend a bit of time considering *melody*. Also, a bit of a break from analysis might be in order as we consider the other side of the coin - *composition*.

Within the realm of *harmony*, we've touched upon the vertical construction of *intervals*, *triads*, and *7th chords*. We've even explored horizontal *harmonic* movement. …how one *chord* moves to the next. Now, we're going to pull out the pencil and *staff* paper and begin putting single *notes* one after the other in logical, effective, and *musical* sequences.

The collection of concepts of *melodic* construction is what we refer to as *voice-leading*. …how the *melody* of a single line (voice) moves from one *note* to the next. As always, remember that we are dealing with the principals and parameters of music from the common practice period, not the contemporary tunes of Taylor Swift.

Let me first list the parameters that will govern our melodic construction, then we'll add some illustrations and deeper explanations following the list.

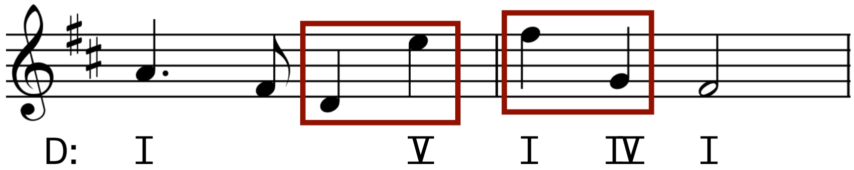
♬ Strong, effective melodic movement (voice-leading) will typically follow these basic principals.

1. Move in step-wise motion or outline the underlying harmony
2. Avoid large skips (7ths and intervals larger than an octave)
3. When approaching a pitch by leap, leave the pitch by step in the opposite direction
4. Avoid augmented interval
5. When writing diminished intervals, resolve the movement by step in the opposite direction
6. Certain tones, referred to as "active" or "tendency" tones, usually resolve in specific ways
    - leading tones tend to resolve UP to the tonic pitch
    - 7ths of chords tend to resolve DOWN by step
    - accidentals tend to resolve (continue) in the direction they are altered
        - flats (lowered pitches) tend to resolve DOWN by step
        - sharps (raised pitches) tend to resolve UP by step

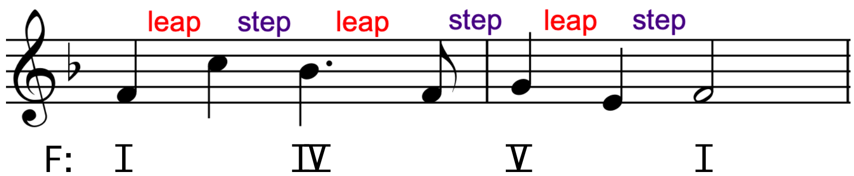
Let's add some illustrations to give these guidelines musical context. The first concept in the list simply gives us the basics of *melodic* writing - **outlining the *harmony*** or **stepping through the *scale***.

♬ We should make a quick note right here. Even though we are exploring *melodic* construction, we should always be aware of the underlying *harmonic* structure and try to avoid *melodic* passages that do not fit within the *harmony*. As we move forward, we'll certainly expand our parameters, but for now, the boundaries are relatively snug.

The second point from our list above indicates that we should **avoid large leaps** in the *melodic line*. Having been a singer and choir director throughout my career, I've always been aware of difficult vocal passages. It seems that the larger the *interval* (other than an *octave*), the more difficult it is to land on the right *note*. One of the practices that I have incorporated into my *melodic* writing is to sing what I have written. If the line is difficult to sing, I usually assume the *voice-leading* could be improved. All of this being said, let's avoid large leaps as we construct our lovely, unforgettable tunes. Here's an example of large leaps hampering good *voice-leading*.

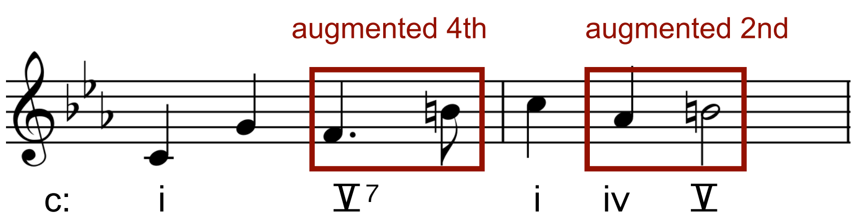


There's another issue with this *melody* above. Following the large leaps, the *melodic* line keeps going in the same direction as the leap. **Following any leap**, it is much better to **step back in the opposite direction**.



In a *minor key* that utilizes the raised $\hat{7}$, there is the possibility of running into a couple of ***augmented melodic intervals***. From $\hat{4}$ up to $\hat{7}$ is an *augmented 4th*. (Note: In a *major key* it's also an *augmented 4th*.) From $\hat{6}$ up to $\hat{7}$ is an *augmented 2nd*.

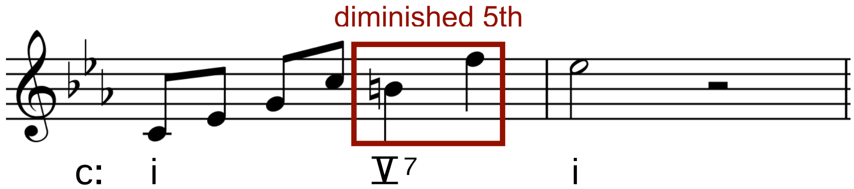
The issue with the *augmented 4th* ($\hat{4}$ up to $\hat{7}$) is that when those two *pitches* are used together they are typically "*active tones*" that need to *resolve* in a specific manner, which is contrary to good *voice-leading*. In the example below, when the *melody* leaps from $\hat{4}$ up to $\hat{7}$, the best *voice-leading* would have it step back down in the opposite direction, but the $\hat{7}$ is the *leading tone* that really "wants" to continue on up to the *tonic*. If the *melody* were to go from $\hat{7}$ down to $\hat{4}$, the $\hat{4}$ would sound like the 7th of the V⁷ which tends to resolve down to $\hat{3}$ instead of stepping back up in the opposite direction of the leap. (These tendencies are much more apparent when listening to the music, compared to just looking at the *notes* on the *staff*.)



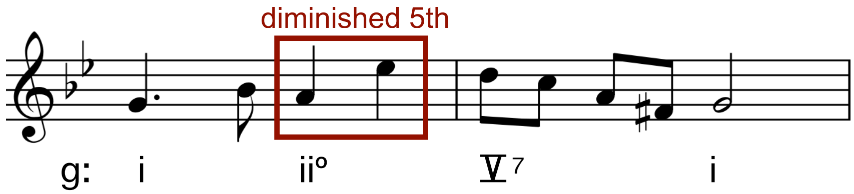
We should remember that *augmented intervals* are deceiving. In the example above, the A♭ to the B♮ certainly looks like a *2nd*, but will sound like a *minor 3rd*. This will be the case with every *augmented interval*. An *augmented 5th* will sound like a *minor 6th*. An *augmented 3rd* will sound like a *perfect 4th*, and so on. Avoiding these points of deception in our *melodic* writing will be a positive move.

Oddly, when *augmented intervals* are *inverted* they become *diminished* and have an acceptable resolution. The *augmented 4th* in the example above will become a *diminished 5th* when *inverted*. The example on the top of the next page demonstrates the resolution for the *diminished fifth*.

The acceptable resolution for a ***diminished melodic interval*** is to step back in the opposite direction.



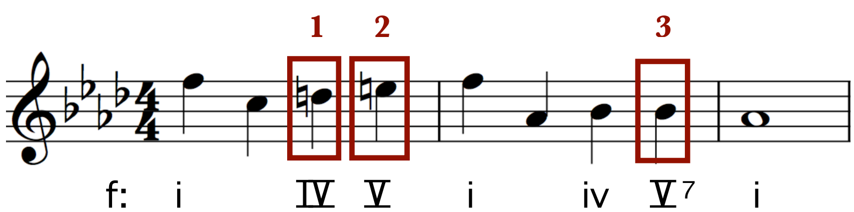
Here's one more example with a *harmony* other than the V^7.



Let's wrap up this section by talking about ***active tones***. Those are the *pitches*, in a *tonal* setting, that have a tendency to *resolve* is specific ways. We've already touched on *leading tones* and *7ths of chords*, but let's reemphasize their resolutions. *Leading tones* have a strong tendency to *resolve* up to the *tonic pitch*. Their very name tells us they have a directional focus. They "lead" us to *tonic*.

Likewise, the *7ths* of *chords* tend to *resolve* down by *step*. One example is the V^7. The *7th* in that *chord*, which is the $\hat{4}$, tends to resolve down to $\hat{3}$. (More often than not, that's the *3rd* of the I.)

Accidentals are the last group of our *active tones*. Any *pitch* that is altered by an *accidental* will tend to *resolve* in the direction it has been altered. A *pitch* that is *sharped* (or raised by a *natural*) will tend to continue on up. Likewise, a *pitch* that is lowered by a *flat* (or a *natural*) will tend to continue on down. Here's one example that includes each of the active tones. (**1** = accidental; **2** = leading tone; **3** = 7th)



As I mentioned, most of these *melodic* movements, tendencies, and *resolutions* can be better understood through listening. Take the opportunity to play each of these examples, with the *harmony* indicated, and make note of how they sound. If you are using the e-book version, you should be able to access the audio examples provided for each illustration.

I need to add one more quick note before we move on. Currently in our study, the only *accidentals* we will run into will be the raised $\hat{6}$ and $\hat{7}$ in *minor*. Other types of *accidentals* won't show up until later.

The next section takes us a little further into the wonderful and fascinating world of *composition*. We'll be looking at how to combine multiple *melodic* lines and how they should interact with each other.

Good times!

Section 23

Harmonic Progression Part-Writing

In this section, we'll examine how simultaneous *melodic lines* should behave with each other. …how multiple linear *voices* interact within a vertical structure. We are going to add new tools to our *compositional* toolbox that will enhance our ability to write *music* that is effective both horizontally and vertically.

Traditionally, in the study of *music theory*, the principals of *part-writing* have been presented in *four-part chorale* style. In this structure, the parts are referred to in the familiar terms of *Soprano, Alto, Tenor, and Bass (SATB)*. These principals of *chorale* construction, however, are easily translated to virtually every medium of *composition*. A note should be made here… Although we are using the labels associated with *singers*, we will not limit our considerations to *vocal performance*. Any and all instruments can be utilized in our four-part arrangement.

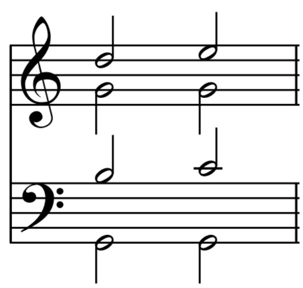
Again, let me note that we are currently working within the framework of the *common practice period*. We are also using, as our starting point, the traditional *four-part chorale* construction. As we move forward, our parameters will expand and adapt, moving past the *common practice period* and beyond four parts into other *compositional* combinations.

♫ Because so much attention has been paid to four-part textures, two widely-accepted labels are used. They are:

- CLOSE structure: an <u>*octave* or less</u> between the *Soprano* and *Tenor*
- OPEN structure: <u>more than an *octave*</u> between the *Soprano* and *Tenor*

CLOSE

OPEN



We've encountered these concepts previously in our study, but here we have more definitive descriptions, ones that relate to the *chorale-type* construction we are currently considering.

When writing in this format, there are a few guidelines that will be necessary for smooth and strong interaction between the parts. As in the previous section, let's first make note of these and then expand them afterwards with illustrations.

1. Nothing should be written above the *Soprano* or below the *Bass*
2. The *Alto* and *Tenor* parts can cross, but only briefly, and only for reasons of strong *voice-leading*
3. Avoid more than an *octave* between adjacent upper voices
    - No more than an *octave* between the *Soprano* and *Alto*
    - No more than an *octave* between the *Alto* and *Tenor*
    - More than an *octave* between the *Tenor* and *Bass* is acceptable
4. Avoid parallel *5ths* and *8ves*
5. Avoid direct *5ths* and *8ves*
6. The strongest movement between voices is <u>contrary motion</u> (going in the opposite direction)

7. An effective movement between voices is oblique motion (one stationary and one moving)
8. Similar motion is acceptable (if avoiding parallel and direct *5ths* and *8ves*)

Often it is necessary to double or omit certain *chord* members to maintain strong *voice-leading*. When doing so, a few guidelines should be considered.

1. When needed, try to double the *root* or *3rd* (double the *5th* only if strong *voice-leading* prohibits doubling the *root* or *3rd*)
2. Do not double *active (tendency) tones*
3. Omit the *5th* if necessary
    - Omitting the *root* changes the *chord* entirely
    - Omitting the *3rd* or *7th* alters the quality of the *chord*

Obviously, if we were writing a piece for band or orchestra, some of these *chorale-oriented* guidelines would not be appropriate. So, keeping our four-part construction in mind, let's talk about the implications of these parameters.

With only four *voices* being heard, the top *voice* (*soprano*) is typically given the main *melodic* material. For this *hymn-like* format, we have all been conditioned to hear the uppermost *line* as the *melody*. That being the case, if the *alto* line moves above the *soprano*, the *alto notes* are automatically interpreted as the *melody*. To maintain the integrity of the *melodic* content in the *soprano*, nothing should be written above it.

The same type of situation applies to the *bass*. That *line* is structural because it serves as the foundation for the *harmony* above it. If you build a house, the foundation must be strong and stable in order to support the construction resting on it. If the *tenor voice* moves below the *bass*, the *bass* loses its status as the lowest-sounding pitch and the foundation becomes unstable. To maintain the integrity and stability of the foundation in the *bass*, nothing should be written below it.

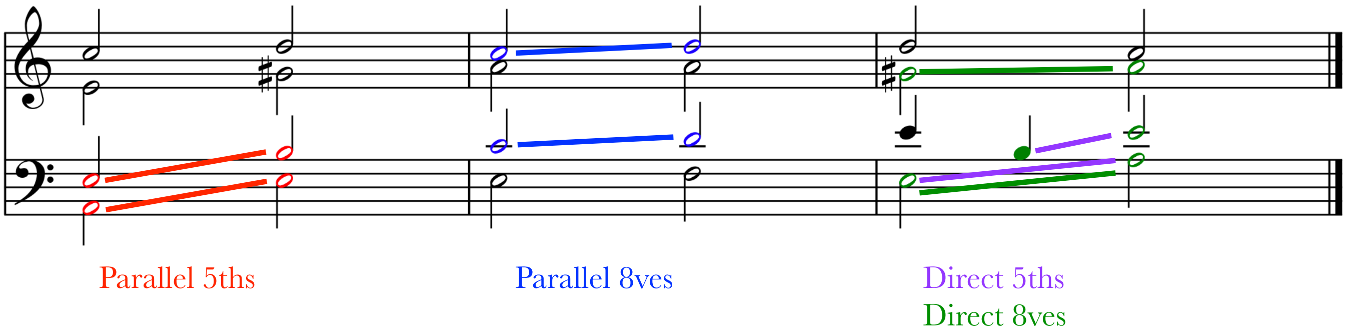
Unlike the outer voices (*soprano* as the highest, *bass* as the lowest) the *alto* and *tenor voices* can easily be crossed without changing the basic structure of the *harmony*. Of course, the *harmonic structure* is not the only issue to consider. The integrity and independence of these two *lines* needs to be guarded. If the *alto* and *tenor* cross, it should not be for long, and it should only happen to maintain strong *voice-leading* for one or both of them.

In *chorale* construction, the *harmony* sort of exists as a unit or unified object. When one of the upper *voices (SAT)* becomes isolated from the other two, this unified arrangement is in jeopardy. It is for that reason that we should not write more than an *octave* between the *soprano* and *alto*, or between the *alto* and *tenor*.

So, you're asking… Why is it different with the *bass*? If you remember, when we first dealt with *figured bass*, we talked about the *continuo*. That practice of having the *bass line* played by one *instrument* and the *harmony* played by a *keyboard instrument* has, in many respects, carried over into our practice of *part-writing*. Even though the *bass* is part of the harmony, it sort of stands alone as the foundation for the other parts. So, if it is separated by more than an *octave* from the *tenor* part, it is acceptable. There is also an acoustical consideration that could be discussed, if we had the time and space. We'll save that for another time. …or class. …or professor.

If you take a look back through *music history*, you'll discover that the practice of *parallel movement* changed through the eras. In the *common practice period*, *parallel and direct 5ths and 8ves* were not acceptable *harmonic movements*. Skip ahead a few years to Debussy and discover that his technique of *planing* (extended *parallel movement* of whole *chords*) was well-accepted and influential. Since we are currently working through that earlier period, let's take a look at what *parallel movements* we need to avoid.

The musical selection below gives us several examples to consider.

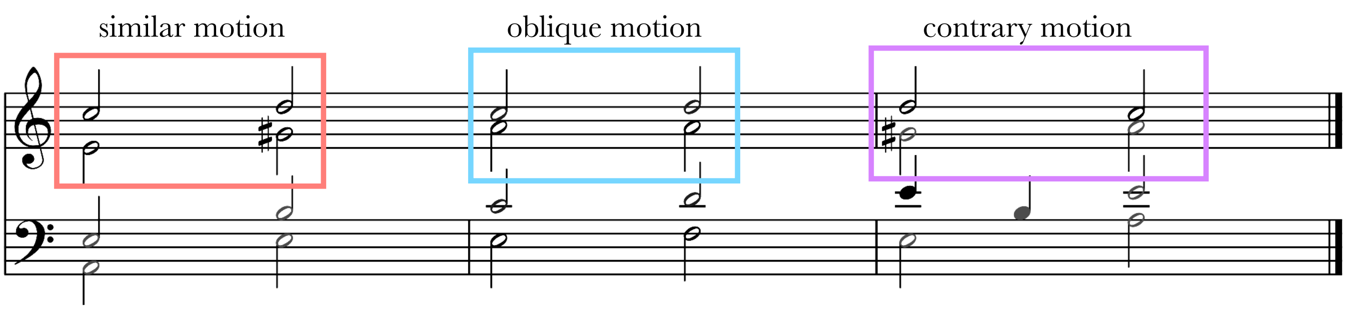


Parallel 5ths happen when two *voices*, a P5 apart, move directly to another P5. If either of the two *intervals* is not a P5, the *movement* is not considered parallel, therefore, it is acceptable.

It's the same situation with *parallel octaves*. If either of the two *intervals* is not a P8, then no *parallel movement* should be noted.

Direct 5ths is when two *voices* move in the same direction to form a P5. It's the same situation with *direct octaves*. In the strictest interpretation, these should be avoided between any two of the four *voices*. From a more relaxed perspective, these *direct intervals* are only a concern when they occur between the two *outer voices* (*soprano* and *bass*).

In addition to *parallel movement*, there are three other types of *movement*. Each of these is more effective than any *direct* or *parallel movement*. Let's use the same *musical selection* to consider these.



The guidelines for doubling and omitting *chord members* are pretty straightforward. Graphics to illustrate those are probably not necessary. Dealing with those will best be considered in a worksheet/homework format.

Let's put a period on this section and move on to those notes that do not fit the harmony (Non-Harmonic Tones) in the next section.

Section 24

Harmonic Progression
Non-Harmonic Tones

We have been considering how *pitches* are used to build horizontal *melodic lines* and vertical *harmonies*, and how they move in succession. In this section we'll be looking at *pitches* that don't belong to the *chords*. …*pitches* that are not heard as members of the prevailing *harmony*. These are what we call *non-harmonic tones* (NHT).

♫ The common *non-harmonic tones* are usually identified as belonging to one of eight categories. Each is categorized and labeled based on two criteria:

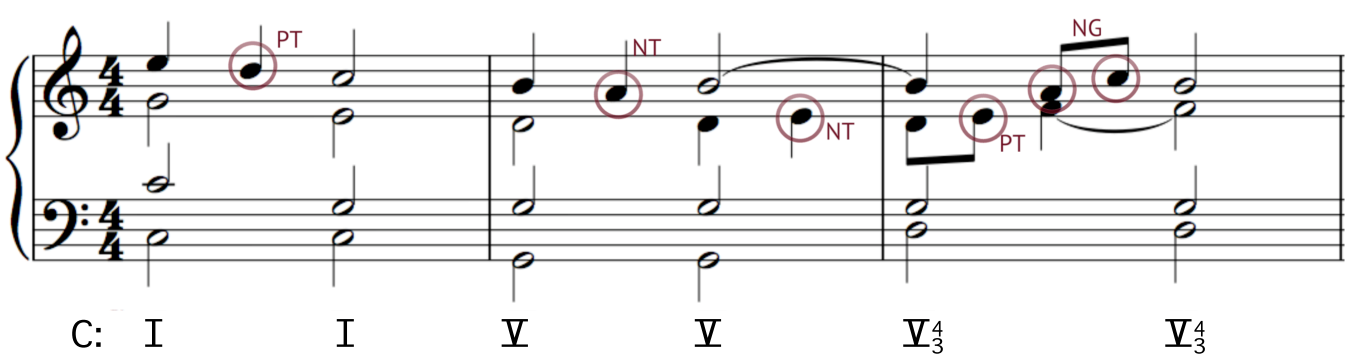
- How it is approached (from the *pitch* that precedes it)
- How it is resolved (to the *pitch* that follows it)

We have now been given another opportunity to create our favorite organizational structure, a table. In the far left column you'll see the name of the *non-harmonic tone* with its label in parentheses. The columns to the right of that will indicate how that *tone* is approached and how it is resolved.

NON-HARMONIC TONE	APPROACHED BY	RESOLVED BY
Passing Tone (PT)	step	step in the same direction
Neighbor Tone (NT)	step	step in the opposite direction
Escape Tone (ET)	step	leap (usually in the opposite direction)
Anticipation (Ant)	step	same pitch
Appoggiatura (Ap)	leap	step (usually in the opposite direction)
Suspension (Sus)	same pitch	step down
Retardation (Ret)	same pitch	step up
Pedal Point	same pitch	same pitch
Neighbor Group (NG)	step	leap a 3rd in the opposite direction then step back to the chord tone

A *Neighbor Group* is a variation/extension of the simple *Neighbor Tone*. The NG is a lower neighbor (below the *chord tone*) and an *upper neighbor* (above the *chord tone*) in combination. The order of the upper and lower *neighbors* can go either direction (below then above or above then below).

As always, it is easier to grasp these concepts when we can see them in a musical context. Immediately below are examples of PT, NT, and NG.

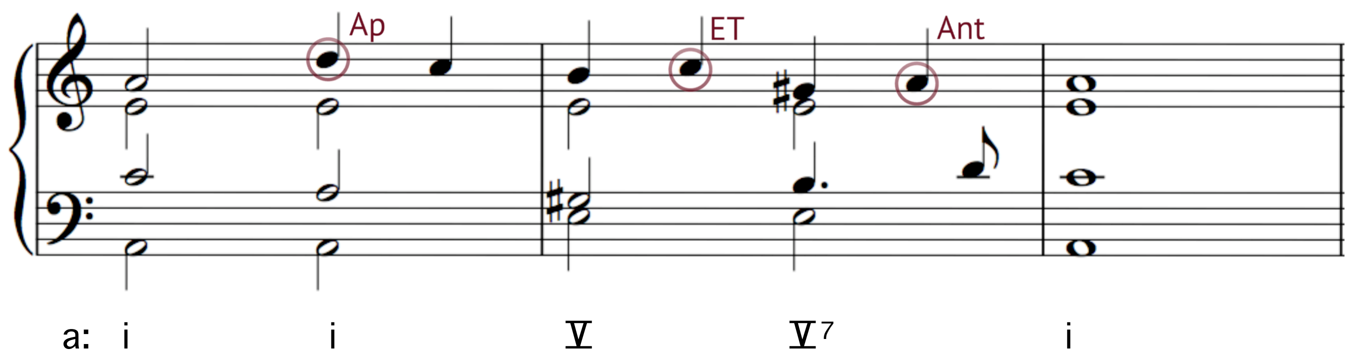


In the top *voice* (*soprano*) of the first *measure* we see a D that moves *step-wise* between the E of the I to the C of the I. The D does not belong to the *harmony* (I). Since it is approached by a step and resolved by a step in the same direction it is considered a PT.

In the second *measure* the A in the *soprano* steps down from the B of the V, then steps back up to the B of the V. In the *alto voice*, the last *note* in that same *measure* is an E that steps up from the D then steps back down to the D of the V on the *downbeat* of the third *measure*. Both of those step away from the *chord tone*, then step back to the same *chord tone*. That is the typical approach and resolution of a NT.

In the third *measure* we see a PT in the *alto* that passes between the 5th of the V to the 7th of the V. The *soprano* of that same *measure* shows us an example of a NG. The *chord tone* is a B, the 3rd of the V. The NG gives us the A below the B, then leaps up a 3rd to C, a step above the B, then resolves back down to the B.

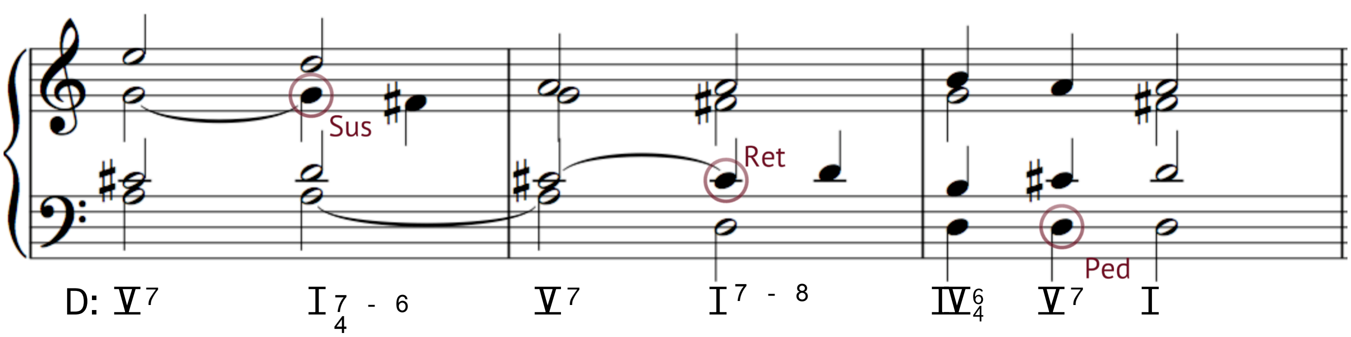
The excerpt below gives us examples of Ap, ET, and Ant.



It should be noted at this point, sometimes the NHTs occur between the *chords* and sometimes they show up with the *chord*. When the NHT is between the *chords* it is considered unaccented. When it occurs at the same time as the *chord*, it is referred to as accented. In the excerpt above, the *appoggiatura* (Ap) is accented (on the third *beat* with the *chord*) while the *escape tone* (ET) and the *anticipation* (Ant) are unaccented, between the *chords*.

The Ap and ET are basically opposites. The Ap is a leap then step. The ET is a step then leap. I would speculate that the ET is usually going to be a step up, followed by a leap down. Likewise, the Ap will usually be found as a leap up then step down. Even if those movements are the norm, the opposite arrangements might be found.

The musical example below shows us the three NHTs that have a stationery *pitch* as part of the mix.



If you'll notice, each of these three NHTs have a *pitch* that hangs on while the *harmony* changes. The *suspension* (Sus) and *retardation* (Ret) have a *pitch* which doesn't move when the *chord* changes. That *pitch* stays in place and resolves after the new *harmony* is heard.

In the first *measure* the *alto* has a G, the 7th of the V^7, which does not change when the *harmony* progresses to the I . It is sustained until the *beat* after the I chord is heard. It then resolves down from the G to the F♯, which is the 3rd of the I. That I is in *second inversion.* You'll notice the *figured bass*, on the third *beat*, indicates there is a 7th and a 4th above the *bass*. The 7th above the *bass*, which is the Sus, then resolves down to a 6th, giving us the typical $^6_4$ *inversion*.

In the second *measure*, we see the Ret. It has the same approach as the Sus but instead of resolving down by step, it resolves up by step. The *figured bass* tells us that the 7th above the *bass* resolves up to an 8va.

The last NHT to look at is the *pedal point* (Ped). Simply put, the Ped is a *pitch* that stays the same as the *harmony* changes. This NHT is typically found in the *bass*, as the lowest-sounding *note,* with the *harmony* changing above it. On occasion, though, it can be found as an internal component with the *harmony* changing above and below.

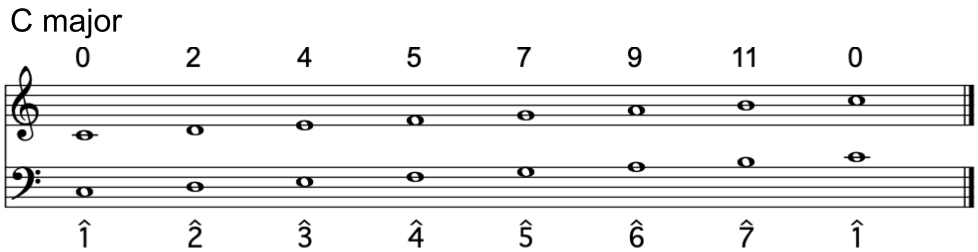
Section 25

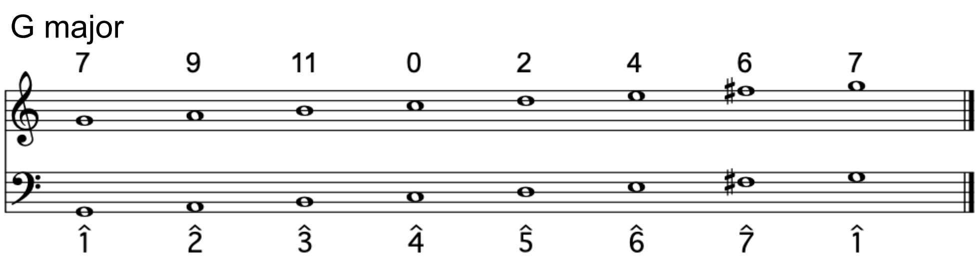
Closely-Related Keys

When we talk about *keys* we are basically talking about groups or sets of *pitches*. If you'll remember from Section 10, we said, "A *key* is basically the collection of *notes* included in the *scale.*" In this section we're going to consider how those collections/sets overlap and relate to each other.

Pitches in any given *key* are identified by their *scale degrees*. In a traditional *diatonic* setting we label these from $\hat{1}$ to $\hat{7}$, with a circumflex above each number. As you remember, each and every *pitch* also has a number from our CPS (Chromatic Pitch Set). For this section, the CPS numbers are very important. We'll focus on those.

Let's look at a couple of scales with their CPS numbers.





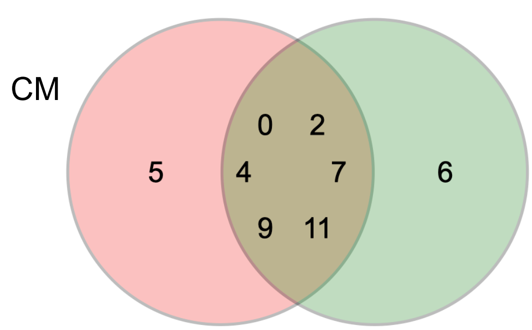
Let's take the CPS numbers (on the top of each staff) and show them as a set. If we designate CM as C major and GM as G major, the sets would look like this:

CM = {0,2,4,5,7,9,11} GM = {7,9,11,0,2,4,6}

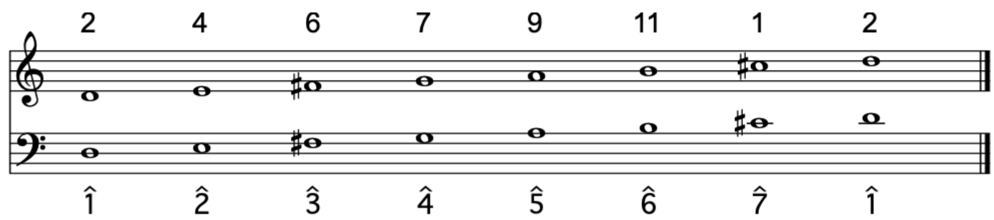
You'll notice that CM is in numeric order, since the *scale* itself starts on 0. Let's put GM in numeric order and compare it with CM.

CM = {0,2,4,5,7,9,11}
GM = {0,2,4,6,7,9,11}

You'll notice that there is only one pitch different. In CM there's a 5 and in GM there's a 6. If you remember how a Venn diagram works, here's what these two sets would look like if they were overlapping.



Here's the D major scale:

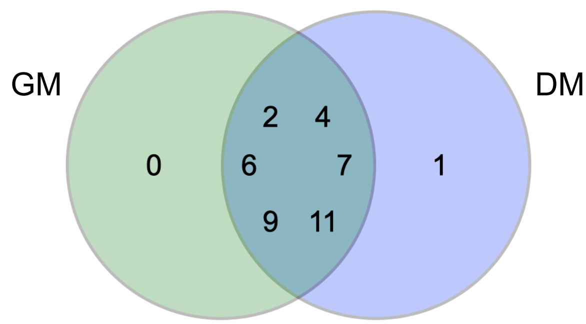


If DM = D major, the set would be: DM = {2,4,6,7,9,11,1}

Let's put them in numeric order and compare GM to DM.

GM = {0,2,4,6,7,9,11}
DM = {1,2,4,6,7,9,11}

Again, let's see what a Venn diagram would look like for these two *key* sets.



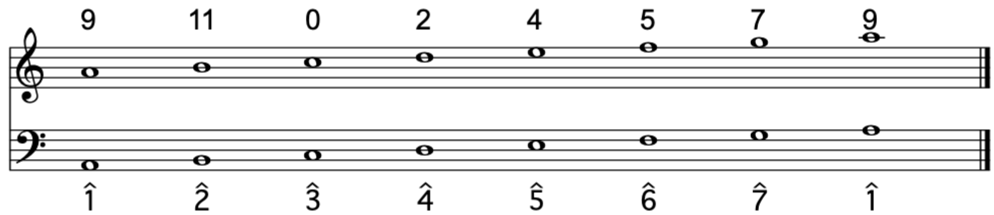
♫ I'm sure you've figured it out. ***Closely-related keys* are ones that have only one *pitch* different.** They have 6 of their 7 *pitches* in common.

The Venn diagram for CM and GM indicates that the intersection of the two keys is the following set: {0,2,4,7,9,11}.   The intersection of GM and DM is: {2,4,6,7,9,11}. In mathematical shorthand these would be:

CM ∩ GM = {0,2,4,7,9,11}   GM ∩ DM = {2,4,6,7,9,11}

It seems like we've pretty well covered this concept… except… What about *minor* keys? We should probably throw a few of those into the mix to see what happens.

Let's take C major and compare it to its relative *minor*. Here's A pure minor.



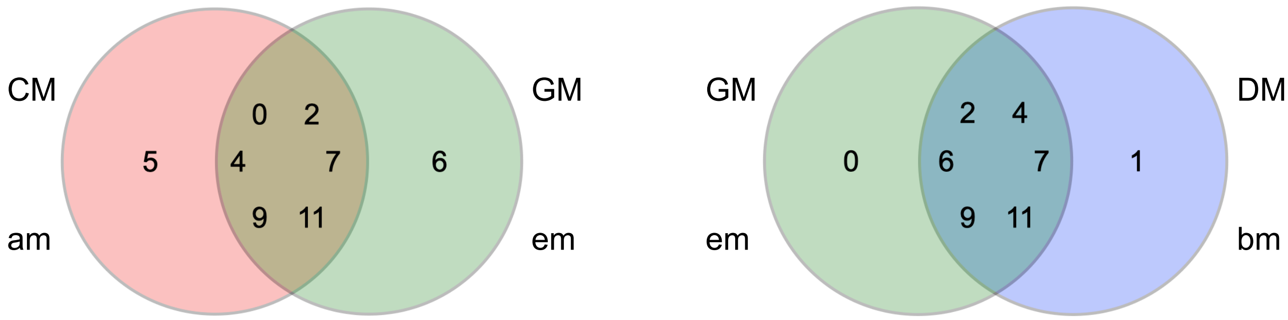
Why don't we give A minor the designation of am !? Here are their two *pitch* sets:

CM = {0,2,4,5,7,9,11}   am = {9,11,0,2,4,5,7}

As before, let's put am in numeric order and compare it to CM.

$$CM = \{0,2,4,5,7,9,11\}$$
$$am = \{0,2,4,5,7,9,11\}$$

Yes. As we've said so many times before, a *major key* and its *relative pure minor* have exactly the same *pitches*. So, let's update the Venn diagrams for our two examples.

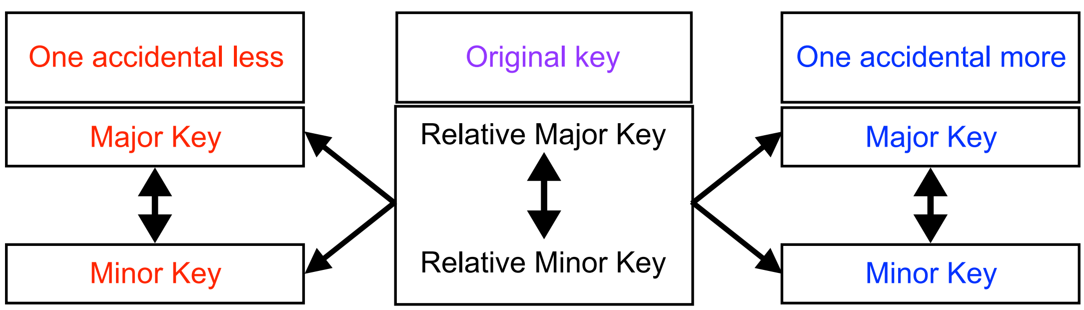


Why don't we go ahead and include the mathematical equations for the *minor key* intersections!?

$$am \cap em = \{0,2,4,7,9,11\} \qquad em \cap bm = \{2,4,6,7,9,11\}$$

Yes. These are exactly the same as their *relative major keys.*

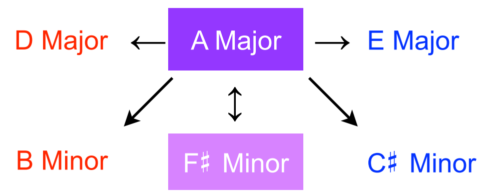
Now that we've looked at the mathematical connections, let's refocus on just the musical ones. If we stay true to our typical style, a graph will probably fit the bill.



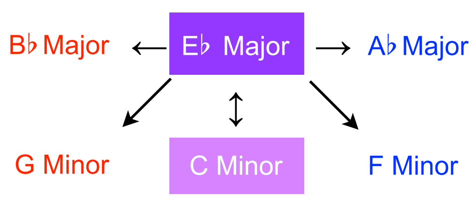
What is it we're actually looking at in this graphic above? Your original key, seen in the middle above, is whatever key you're starting in. If it's *major*, it has a *relative minor* and vice-versa. If you subtract one *accidental* from the *key signature* you have a closely-related *major key* and *minor key* (on the left). If you add one *accidental* to the original *key signature* you have another set of *closely-related keys*, one *major* and one *minor* (on the right).

You're probably asking, "What does it mean to add and subtract *accidentals*?" Let's look at an example.

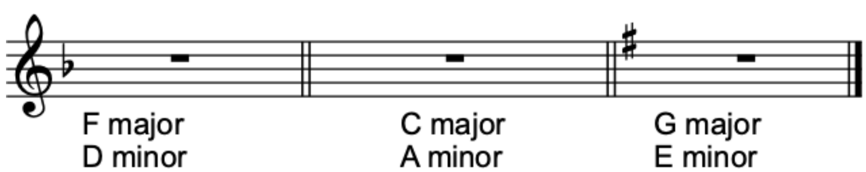
If the original *key* is A major (F♯ minor) with 3 *sharps* in the *key signature*, adding 1 *accidental* to the *key signature* would take us to E major (C♯ minor). Subtracting 1 *accidental* would take us to D major (B minor). So, from A major there would be a total of 5 other *keys* (including the *relative minor*) that would be closely-related. Let's see that in a graph, like the one on the previous page.



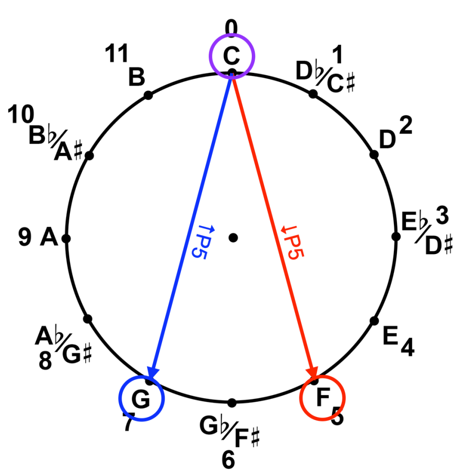
Let's consider another example.

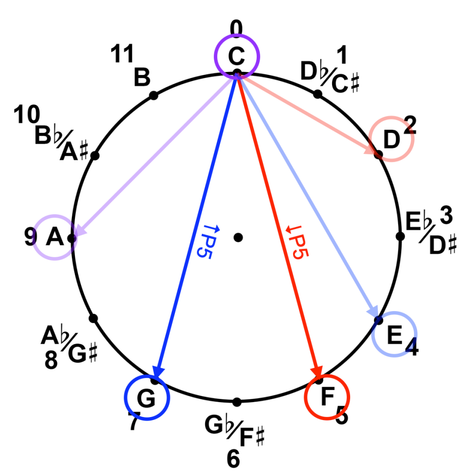


You are probably ready to ask, "What about C major? How would that work?" Here's the answer in musical form.

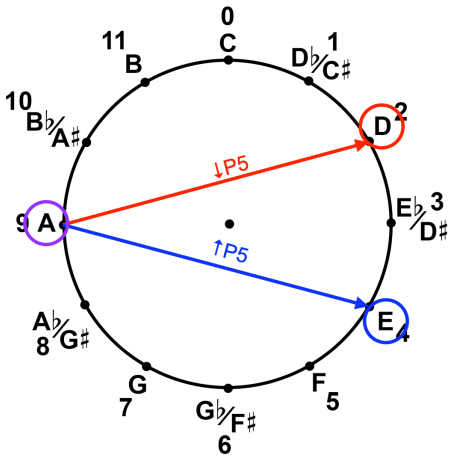


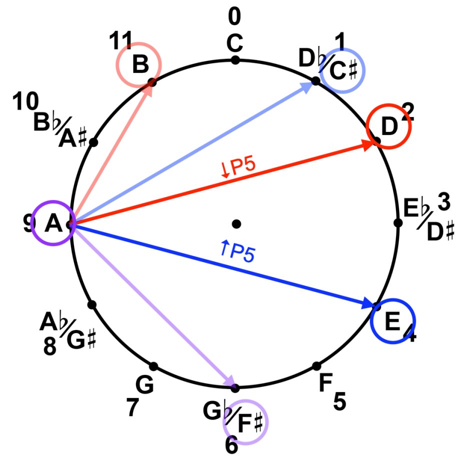
I know it's been a while, but let's put these examples on the CPS to see how they look and to see if there are other relationships we can discover. The example on the left below is from the original *key signature* of C Major to its *closely-related keys*. The right-hand example includes the *relative minors* of each.





Here's one more example with A Major being the original key.





If you carefully study these CPS graphs you'll discover something really interesting. Take A Major (above) as our starting *key*. The *closely-related major keys* are actually the *keys* based on the IV and V *chords* of A Major. The *relative minors* of these two are *keys* based on the ii and iii chords of A Major, and, of course, the *relative minor* of our starting *key* is based on the vi.

Let all of that sink in for a moment. Gather up the *diatonic chords* connected to our *closely-related keys* and what you have is this… keys based on the I, ii, iii, IV, V and vi chords. So, from our starting point of A Major we'd have this:

I	ii	iii	IV	V	vi
A Major	B Minor	C♯ Minor	D Major	E Major	F♯ Minor

As with all of our concepts of this nature, it will work in any *key*. Here are two more examples:

I	ii	iii	IV	V	vi
C Major	D Minor	E Minor	F Major	G Major	A Minor

I	ii	iii	IV	V	vi
E♭ Major	F Minor	G Minor	A♭ Major	B♭ Major	C minor

Before we put this section to bed, let's consider a starting *key* that's *minor*.

If your original key is *minor*, all of the same relationships will be in tact, just shifted a bit. Let's take A Minor (pure form) as our example. Knowing that it is the *relative minor* of C Major, here's what we have:

i	III	iv	v	VI	VII	
A Minor	C Major	D Minor	E Minor	F Major	G Major	
	I	ii	iii	IV	V	vi
	C Major	D Minor	E Minor	F Major	G Major	A Minor

As you can see, all of the same *keys* are closely-related, but the starting point has shifted.

Before we end this we need to take note of a couple of issues. These concepts above will work until you make your way around to C♯ major and C♭ major. Those are the two *keys* that contain all the *sharps* or all the *flats*. What are the issues with these two *keys* (and their *relative minors*)?

In C♯ major, when you try to add a *sharp* to the *key signature*, it will create *keys* that do not exist, G♯ major and E♯ minor. So, what's the problem creating those two *keys*? Since C♯ major contains a *sharp* for every *pitch* in our *diatonic scale*, there are no more *pitches* to add a *sharp* to. For G♯ major and E♯ minor you would end up with a key signature that contained seven *sharps* plus an additional F𝄪 (F-double sharp). …not a good plan.

The same thing happens when you try to add a *flat* to C♭ major (A♭ minor). You end up with F♭ major and D♭ minor. The *key signature* for those would have to include a B𝄫.

So, bottom line… C♯ major (A♯ minor) and C♭ major (A♭ minor) will have two fewer *closely-related keys* than all the other *keys*.

That's it. That's all I have for you at this point.

Worksheets

MOD 7

Name:________________________________ ID#________________

Calculate the congruent number, in MOD 7, for each of the following:

12 ≡ _____ (mod 7)	15 ≡ _____ (mod 7)	11 ≡ _____ (mod 7)
8 ≡ _____ (mod 7)	10 ≡ _____ (mod 7)	9 ≡ _____ (mod 7)
17 ≡ _____ (mod 7)	14 ≡ _____ (mod 7)	18 ≡ _____ (mod 7)
13 ≡ _____ (mod 7)	16 ≡ _____ (mod 7)	19 ≡ _____ (mod 7)

Complete the following equations in MOD 7.

7 + 7 = _____ (mod 7)	6 + 4 = _____ (mod 7)	4 + 5 = _____ (mod 7)
4 + 3 = _____ (mod 7)	7 + 5 = _____ (mod 7)	6 + 6 = _____ (mod 7)
2 + 7 = _____ (mod 7)	3 + 5 = _____ (mod 7)	2 + 2 = _____ (mod 7)
6 + 5 = _____ (mod 7)	6 + 7 = _____ (mod 7)	4 + 2 = _____ (mod 7)

Using our Musical Alphabet, in MOD 7, complete the following equations.

If 1 = D then 3 = ____	If 1 = A then 5 = ____	If 1 = E then 7 = ____
If 1 = B then 2 = ____	If 1 = F then 4 = ____	If 1 = C then 6 = ____
If 1 = G then 5 = ____	If 3 = D then 1 = ____	If 5 = A then 1 = ____
If 7 = E then 1 = ____	If 2 = B then 1 = ____	If 4 = F then 1 = ____
If 6 = C then 1 = ____	If 3 = G then 1 = ____	If 5 = D then 1 = ____

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MOD 12 - 1

Name:________________________________ ID#________________

Calculate the congruent number, in MOD 12, for each of the following:

$12 \equiv$ _____ (mod 12)	$15 \equiv$ _____ (mod 12)	$14 \equiv$ _____ (mod 12)
$19 \equiv$ _____ (mod 12)	$11 \equiv$ _____ (mod 12)	$22 \equiv$ _____ (mod 12)
$17 \equiv$ _____ (mod 12)	$13 \equiv$ _____ (mod 12)	$20 \equiv$ _____ (mod 12)
$24 \equiv$ _____ (mod 12)	$16 \equiv$ _____ (mod 12)	$27 \equiv$ _____ (mod 12)

Complete the following equations in MOD 12.

2 + 9 = _____ (mod 12)	6 + 7 = _____ (mod 12)	3 + 8 = _____ (mod 12)
4 + 10 = _____ (mod 12)	8 - 11 = _____ (mod 12)	2 - 7 = _____ (mod 12)
4 + 8 = _____ (mod 12)	9 + 7 = _____ (mod 12)	5 - 11 = _____ (mod 12)
8 + 7 = _____ (mod 12)	7 - 9 = _____ (mod 12)	8 + 7 = _____ (mod 12)

For each of the pitches given below enter the corresponding number from the CPS.

G♯ = _____	C = _____	E = _____	G = _____	B♭ = _____
G♭ = _____	C♯ = _____	E♭ = _____	A♭ = _____	B = _____
F♯ = _____	D = _____	A♯ = _____	D♯ = _____	D♭ = _____
F = _____	A = _____			

INTENTIONALLY BLANK

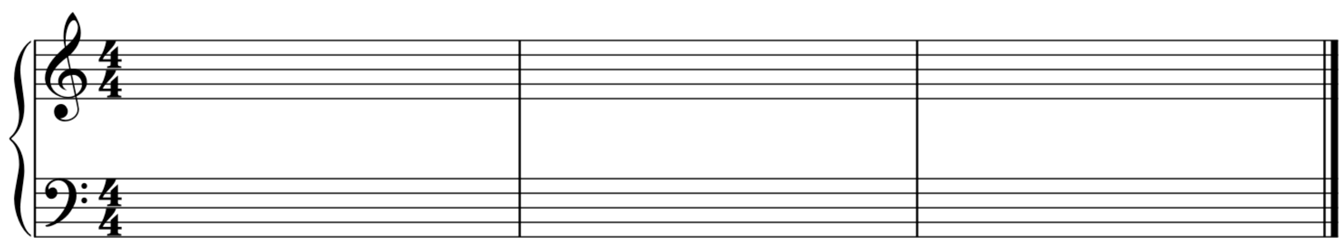
MOD 12 - 2

Name ______________________________ ID# ___________

Referring to the CPS, write your phone number on the music staff below. Use the time signature and rhythm shown in the example below. (This is the Music Office number.)

example

Your phone number



Whose number is this?

Call this one for an automated response.

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Diatonic Intervals - 1

Name:________________________________ ID#________________

Give the diatonic interval for each of the following equations. (For now, these will not have the quality designation, only the distance.)

$\{\hat{1},\uparrow\hat{7}\}$ = _____ (mod 7)	$\{\hat{2},\uparrow\hat{6}\}$ = _____ (mod 7)	$\{\hat{3},\uparrow\hat{5}\}$ = _____ (mod 7)
$\{\hat{1},\downarrow\hat{7}\}$ = _____ (mod 7)	$\{\hat{2},\downarrow\hat{6}\}$ = _____ (mod 7)	$\{\hat{3},\downarrow\hat{5}\}$ = _____ (mod 7)
$\{\hat{2},\uparrow\hat{7}\}$ = _____ (mod 7)	$\{\hat{3},\uparrow\hat{6}\}$ = _____ (mod 7)	$\{\hat{4},\uparrow\hat{5}\}$ = _____ (mod 7)
$\{\hat{2},\downarrow\hat{7}\}$ = _____ (mod 7)	$\{\hat{3},\downarrow\hat{6}\}$ = _____ (mod 7)	$\{\hat{4},\downarrow\hat{5}\}$ = _____ (mod 7)
$\{\hat{3},\uparrow\hat{7}\}$ = _____ (mod 7)	$\{\hat{4},\uparrow\hat{6}\}$ = _____ (mod 7)	$\{\hat{5},\uparrow\hat{5}\}$ = _____ (mod 7)
$\{\hat{3},\downarrow\hat{7}\}$ = _____ (mod 7)	$\{\hat{4},\downarrow\hat{6}\}$ = _____ (mod 7)	$\{\hat{5},\downarrow\hat{5}\}$ = _____ (mod 7)
$\{\hat{4},\uparrow\hat{7}\}$ = _____ (mod 7)	$\{\hat{5},\uparrow\hat{6}\}$ = _____ (mod 7)	$\{\hat{6},\uparrow\hat{5}\}$ = _____ (mod 7)
$\{\hat{4},\downarrow\hat{7}\}$ = _____ (mod 7)	$\{\hat{5},\downarrow\hat{6}\}$ = _____ (mod 7)	$\{\hat{6},\downarrow\hat{5}\}$ = _____ (mod 7)
$\{\hat{5},\uparrow\hat{7}\}$ = _____ (mod 7)	$\{\hat{6},\uparrow\hat{6}\}$ = _____ (mod 7)	$\{\hat{7},\uparrow\hat{5}\}$ = _____ (mod 7)
$\{\hat{5},\downarrow\hat{7}\}$ = _____ (mod 7)	$\{\hat{6},\downarrow\hat{6}\}$ = _____ (mod 7)	$\{\hat{7},\downarrow\hat{5}\}$ = _____ (mod 7)
$\{\hat{6},\uparrow\hat{7}\}$ = _____ (mod 7)	$\{\hat{7},\uparrow\hat{6}\}$ = _____ (mod 7)	$\{\hat{1},\uparrow\hat{5}\}$ = _____ (mod 7)
$\{\hat{6},\downarrow\hat{7}\}$ = _____ (mod 7)	$\{\hat{7},\downarrow\hat{6}\}$ = _____ (mod 7)	$\{\hat{1},\downarrow\hat{5}\}$ = _____ (mod 7)
$\{\hat{7},\uparrow\hat{7}\}$ = _____ (mod 7)	$\{\hat{1},\uparrow\hat{6}\}$ = _____ (mod 7)	$\{\hat{2},\uparrow\hat{5}\}$ = _____ (mod 7)
$\{\hat{7},\downarrow\hat{7}\}$ = _____ (mod 7)	$\{\hat{1},\downarrow\hat{6}\}$ = _____ (mod 7)	$\{\hat{2},\downarrow\hat{5}\}$ = _____ (mod 7)

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Diatonic Intervals - 2

Name:________________________________ ID#________________

example:
If $\hat{1}$ = D then ↑5th from $\hat{2}$ = ___ (mod 7) ; the answer is B

If $\hat{1}$ = F then ↑2nd from $\hat{3}$ = ___ (mod 7) If $\hat{1}$ = F then ↓2nd from $\hat{3}$ = ___ (mod 7)

If $\hat{1}$ = F then ↑3rd from $\hat{4}$ = ___ (mod 7) If $\hat{1}$ = F then ↓3rd from $\hat{4}$ = ___ (mod 7)

If $\hat{1}$ = B then ↑4th from $\hat{5}$ = ___ (mod 7) If $\hat{1}$ = B then ↓4th from $\hat{5}$ = ___ (mod 7)

If $\hat{1}$ = B then ↑5th from $\hat{6}$ = ___ (mod 7) If $\hat{1}$ = B then ↓5th from $\hat{6}$ = ___ (mod 7)

If $\hat{1}$ = E then ↑6th from $\hat{7}$ = ___ (mod 7) If $\hat{1}$ = E then ↓6th from $\hat{7}$ = ___ (mod 7)

If $\hat{1}$ = E then ↑7th from $\hat{1}$ = ___ (mod 7) If $\hat{1}$ = E then ↓7th from $\hat{1}$ = ___ (mod 7)

If $\hat{1}$ = A then ↑2nd from $\hat{2}$ = ___ (mod 7) If $\hat{1}$ = A then ↓2nd from $\hat{2}$ = ___ (mod 7)

If $\hat{1}$ = A then ↑3rd from $\hat{3}$ = ___ (mod 7) If $\hat{1}$ = A then ↓3rd from $\hat{3}$ = ___ (mod 7)

If $\hat{1}$ = D then ↑4th from $\hat{4}$ = ___ (mod 7) If $\hat{1}$ = D then ↓4th from $\hat{4}$ = ___ (mod 7)

If $\hat{1}$ = D then ↑5th from $\hat{5}$ = ___ (mod 7) If $\hat{1}$ = D then ↓5th from $\hat{5}$ = ___ (mod 7)

If $\hat{1}$ = G then ↑6th from $\hat{6}$ = ___ (mod 7) If $\hat{1}$ = G then ↓6th from $\hat{6}$ = ___ (mod 7)

If $\hat{1}$ = G then ↑7th from $\hat{7}$ = ___ (mod 7) If $\hat{1}$ = G then ↓7th from $\hat{7}$ = ___ (mod 7)

If $\hat{1}$ = C then ↑2nd from $\hat{1}$ = ___ (mod 7) If $\hat{1}$ = C then ↓2nd from $\hat{1}$ = ___ (mod 7)

If $\hat{1}$ = C then ↑3rd from $\hat{2}$ = ___ (mod 7) If $\hat{1}$ = C then ↓3rd from $\hat{2}$ = ___ (mod 7)

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Ascending Intervals - 1

Name:________________________________ ID#________________

What are the labels for each of the following interval types?

Augmented _____ Perfect _____ Diminished _____

Major _____ Minor _____

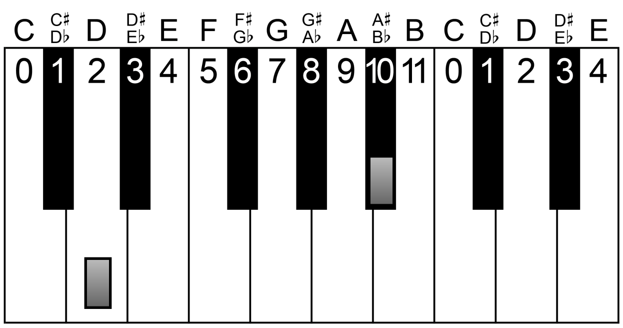
How many half-steps are each of the following intervals?

m6 ____ PU _____ M3 ____ m7 ____ M6 ____

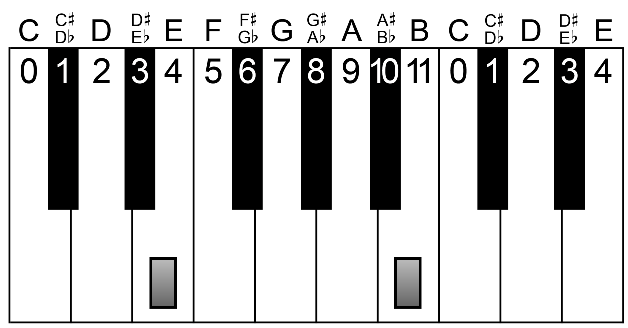
P5 ____ M7 _____ m2 ____ A4 ____ P4 ____

P8 ____ d5 _____ m3 ____ M2 ____

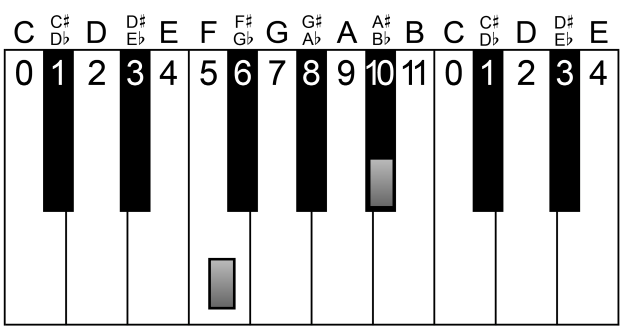
Identify and label each of the following intervals on the keyboard.



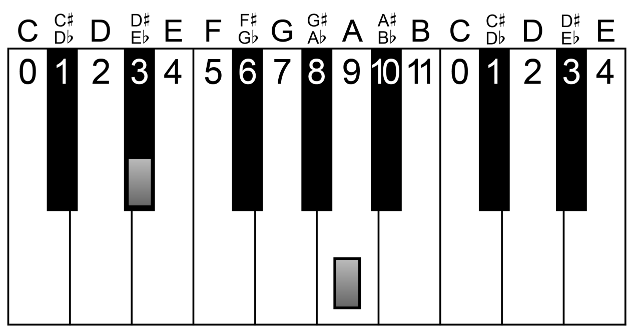
______ interval



______ interval

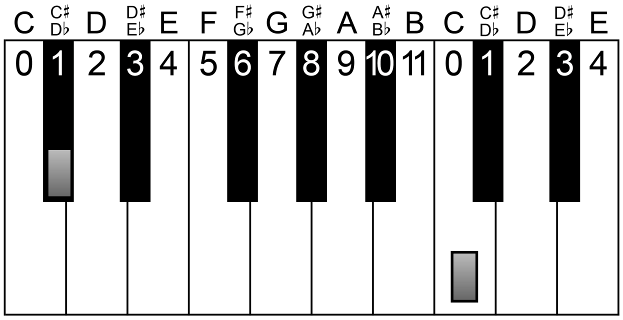


______ interval

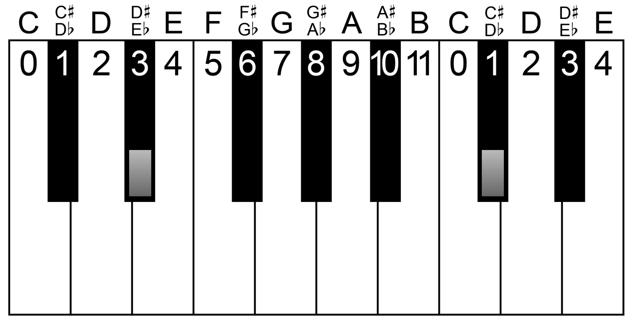


______ interval

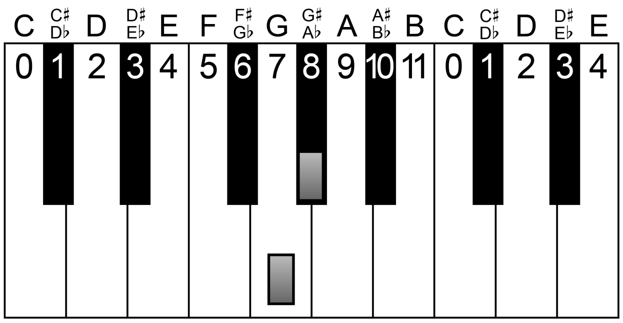
Ascending Intervals - 2



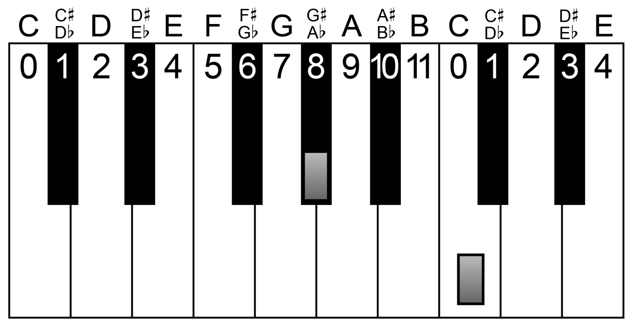
______ interval



______ interval

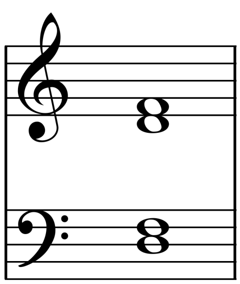


______ interval

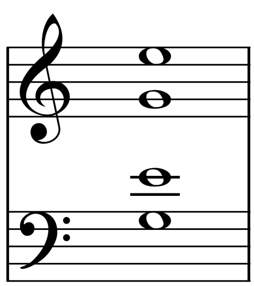


______ interval

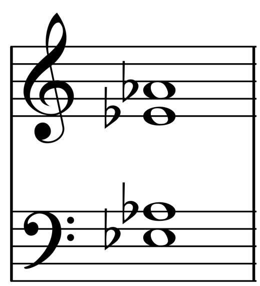
Identify and label each of the following intervals on the staff.



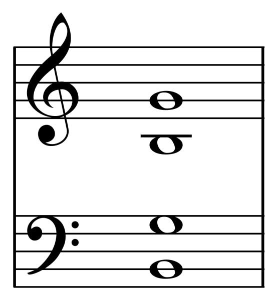
_____ interval



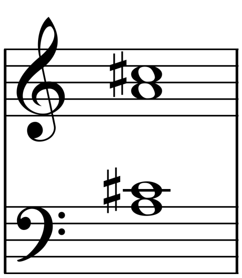
_____ interval



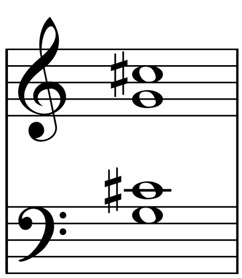
_____ interval



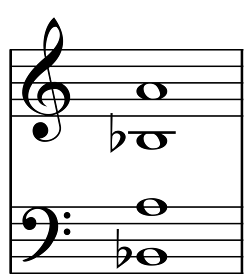
_____ interval



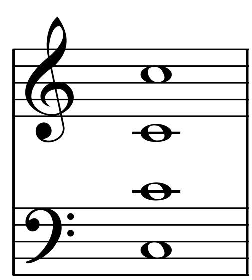
_____ interval



_____ interval



_____ interval



_____ interval

Ascending Intervals - 3

Name:________________________________ ID#________________

Using the chart on page 17 of the text, as well as the CPS, please complete each of the equations below.

Example: P4 up from D

P4 = {2,(2+5)} → P4 = {2,7} → P4 = {D, G}

M3 up from D♭

M3 = { __, (__ + __)} → M3 = { __, __ } → M3 = { __, __ }

m6 up from E

m6 = { __, (__ + __)} → m6 = { __, __ } → m6 = { __, __ }

M2 up from F♯

M2 = { __, (__ + __)} → M2 = { __, __ } → M2 = { __, __ }

m7 up from G

m7 = { __, (__ + __)} → m7 = { __, __ } → m7 = { __, __ }

P4 up from A♭

P4 = { __, (__ + __)} → P4 = { __, __ } → P4 = { __, __ }

P5 up from B

P5 = { __, (__ + __)} → P5 = { __, __ } → P5 = { __, __ }

A4 up from C

A4 = { __, (__ + __)} → A4 = { __, __ } → A4 = { __, __ }

d5 up from D

d5 = { __, (__ + __)} → d5 = { __, __ } → d5 = { __, __ }

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Descending Intervals - 1

Name:_______________________________ ID#_______________

Using the chart on page 21, as well as the keyboard and CPS graphs on page 112, complete the following equations. Give the CPS numbers as well as the letter names.

example: equation If p = 3 then P5↓ = { ___, ___ } (mod 12)
answer 3,8 ; E♭ , A♭

If p = 5 then P4↓ = { ___, ___ } (mod 12) answer ___, ___ ; ___, ___

If p = 4 then P5↓ = { ___, ___ } (mod 12) answer ___, ___ ; ___, ___

If p = 3 then m6↓ = { ___, ___ } (mod 12) answer ___, ___ ; ___, ___

If p = 2 then M6↓ = { ___, ___ } (mod 12) answer ___, ___ ; ___, ___

If p = 1 then m7↓ = { ___, ___ } (mod 12) answer ___, ___ ; ___, ___

If p = 0 then M7↓ = { ___, ___ } (mod 12) answer ___, ___ ; ___, ___

If p = 11 then m2↓ = { ___, ___ } (mod 12) answer ___, ___ ; ___, ___

If p = 10 then M2↓ = { ___, ___ } (mod 12) answer ___, ___ ; ___, ___

If p = 9 then m3↓ = { ___, ___ } (mod 12) answer ___, ___ ; ___, ___

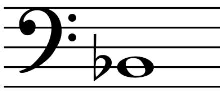
If p = 8 then M3↓ = { ___, ___ } (mod 12) answer ___, ___ ; ___, ___

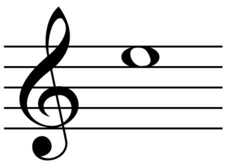
If p = 7 then A4↓ = { ___, ___ } (mod 12) answer ___, ___ ; ___, ___

If p = 6 then d5↓ = { ___, ___ } (mod 12) answer ___, ___ ; ___, ___

Descending Intervals - 2

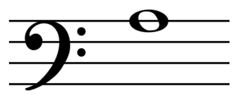
From the pitch given on the staff, indicate the letter names of the given interval.

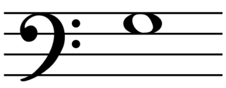
example:  M3↓ = B♭, G♭

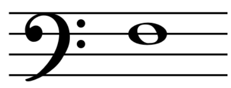
 P4↓ = _____, _____

 P5↓ = _____, _____

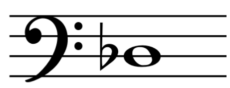
 m6↓ = _____, _____

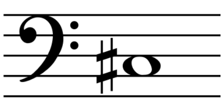
 M6↓ = _____, _____

 m7↓ = _____, _____

 M7↓ = _____, _____

 m2↓ = _____, _____

 M2↓ = _____, _____

 m3↓ = _____, _____

 M3↓ = _____, _____

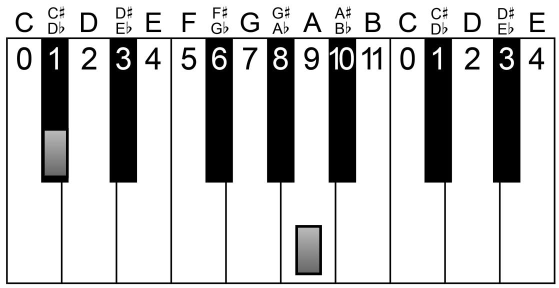
 A4↓ = _____, _____

 d5↓ = _____, _____

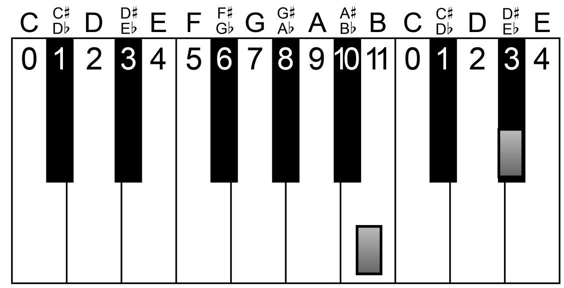
Descending Intervals - 3

Name:________________________________ ID#________________

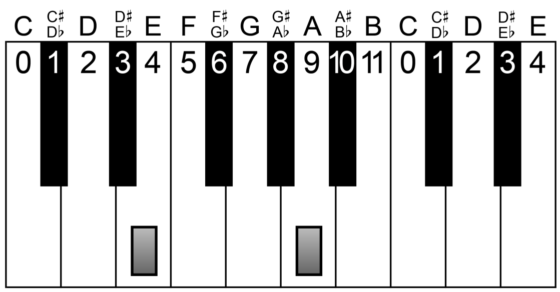
Identify and label each of the intervals on the keyboards below.



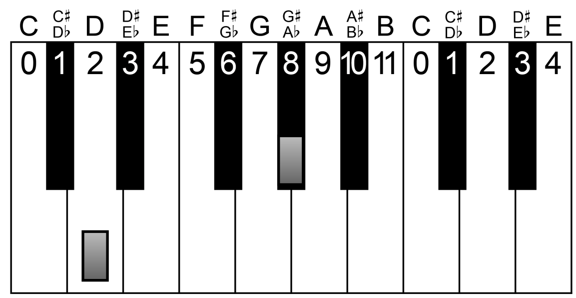
_____ interval



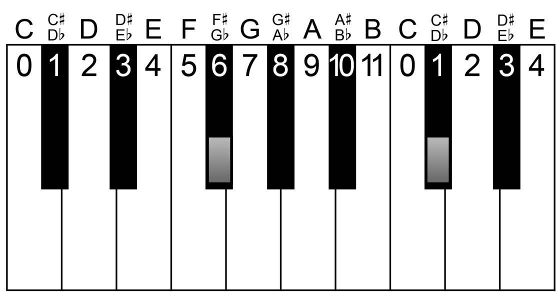
_____ interval



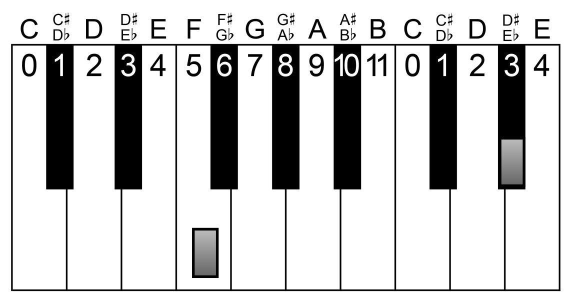
_____ interval



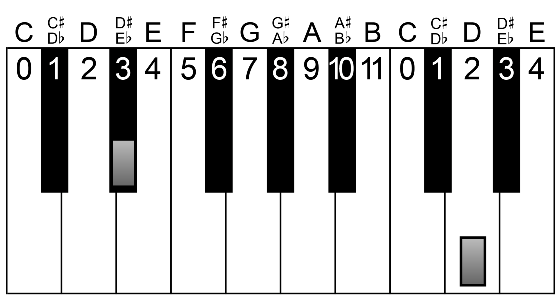
_____ interval



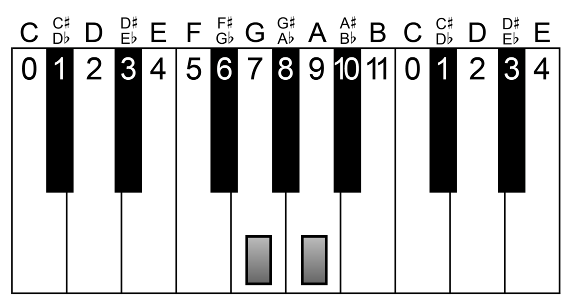
_____ interval



_____ interval



_____ interval



_____ interval

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Interval Inversions - 1

Name:________________________________ ID#________________

Give the complimentary interval (inversion) for each of the following intervals. The original interval will be designated as O and the inversion as I.

example: equation $O = 5 \rightarrow I =$ ___ (mod 12)
answer $I = 7$

$O = 11 \rightarrow I =$ ___ (mod 12) $O = 9 \rightarrow I =$ ___ (mod 12) $O = 6 \rightarrow I =$ ___ (mod 12)

$O = 3 \rightarrow I =$ ___ (mod 12) $O = 1 \rightarrow I =$ ___ (mod 12) $O = 2 \rightarrow I =$ ___ (mod 12)

$O = 4 \rightarrow I =$ ___ (mod 12) $O = 5 \rightarrow I =$ ___ (mod 12) $O = 7 \rightarrow I =$ ___ (mod 12)

$O = 8 \rightarrow I =$ ___ (mod 12) $O = 10 \rightarrow I =$ ___ (mod 12) $O = 0 \rightarrow I =$ ___ (mod 12)

Give the inversions for the following intervals.

example: equation P4↑ → ___
answer P4↑ → P5↓

M7↑ → ____ m6↓ → ____ P5↑ → ____ A4↓ → ____

m3↑ → ____ M2↓ → ____ m7↑ → ____ M6↓ → ____

d5↑ → ____ M3↓ → ____ m2↑ → ____ P4↓ → ____

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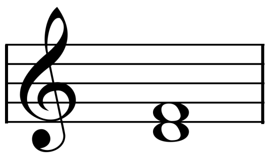
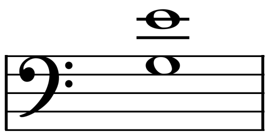
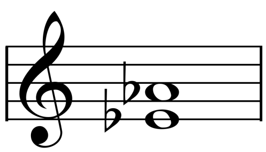
Interval Inversions - 2

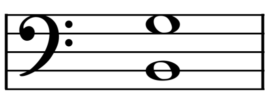
Name:________________________________ ID#________________

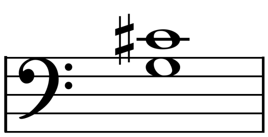
For each of the following, identify the given interval, then give its inversion.

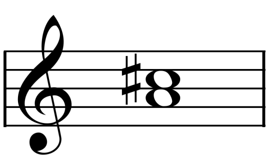
 interval ______ inversion ______

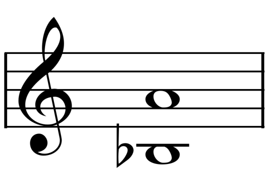
 interval ______ inversion ______

 interval ______ inversion ______

 interval ______ inversion ______

 interval ______ inversion ______

 interval ______ inversion ______

 interval ______ inversion ______

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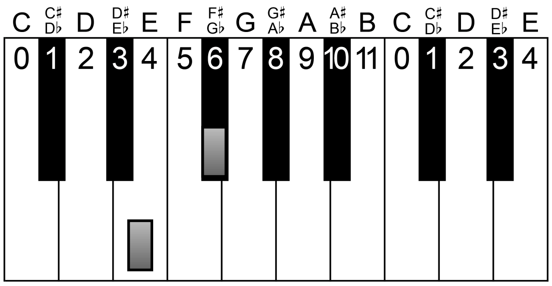
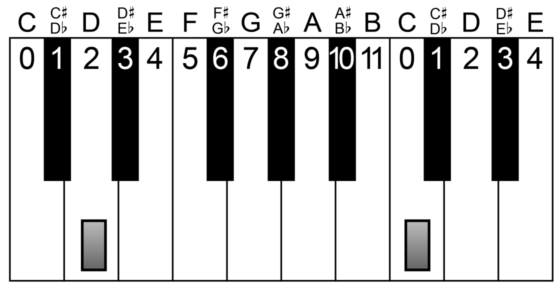
Interval Inversions - 3

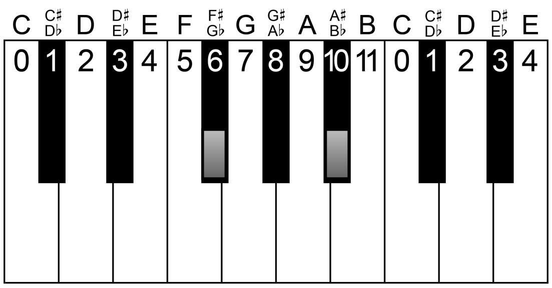
For each of the following, identify the given interval, then give its inversion.



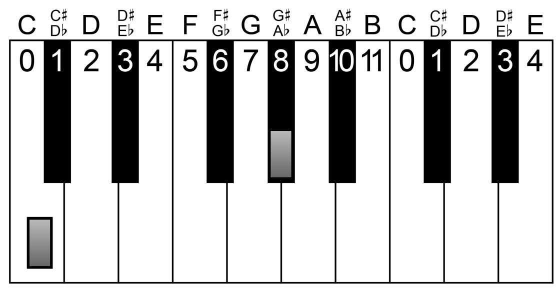
interval ____ inversion ____



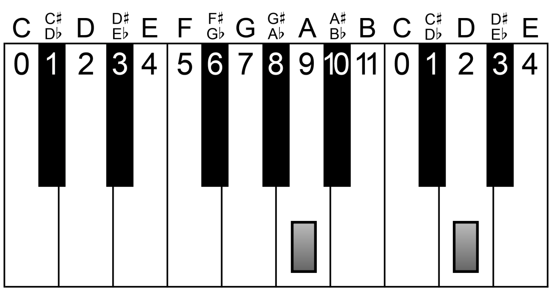
interval ____ inversion ____



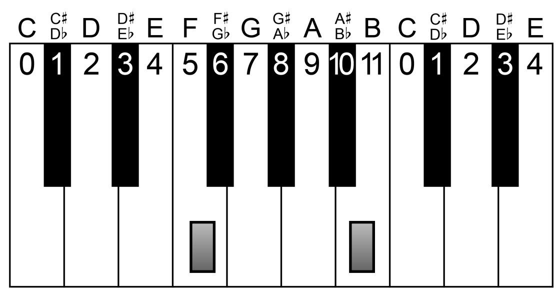
interval ____ inversion ____



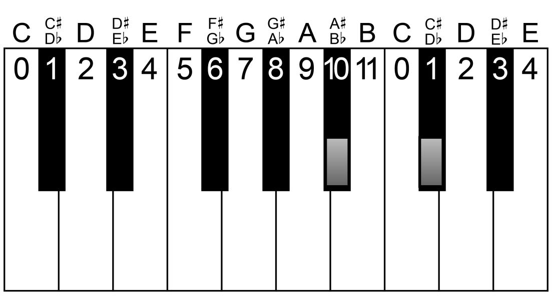
interval ____ inversion ____



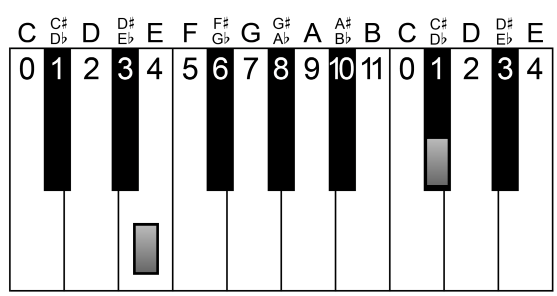
interval ____ inversion ____



interval ____ inversion ____



interval ____ inversion ____



interval ____ inversion ____

Augmented and Diminished Intervals - 1

Name:________________________________ ID#________________

For each of the given note sets, identify the original interval, then add an accidental to one of the pitches to create the intended intervals. Give the letter names.

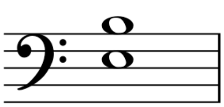
example:

original interval = _____ A4 = ____, ____ or ____, ____

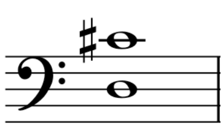
original interval = P4 A4 = D, G♯ or D♭, G

original interval = _____ M6 = ____, ____ or ____, ____

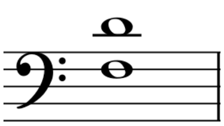
original interval = _____ m3 = ____, ____ or ____, ____



original interval = _____ d5 = ____, ____ or ____, ____



original interval = _____ m7 = ____, ____ or ____, ____



original interval = _____ A6 = ____, ____ or ____, ____

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Augmented and Diminished Intervals - 2

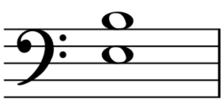
Name:________________________________ ID#________________

original interval = _____ M3 = ____, ____ or ____, ____

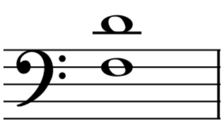
original interval = _____ d7 = ____, ____ or ____, ____

original interval = _____ A4 = ____, ____ or ____, ____

original interval = _____ d6 = ____, ____ or ____, ____



original interval = _____ A5 = ____, ____ or ____, ____



original interval = _____ m6 = ____, ____ or ____, ____

original interval = _____ M7 = ____, ____ or ____, ____

For each of the given note sets below, identify the interval, give the inversion, and give the enharmonic equivalent.
example:

original interval = A2 inversion = d7 enharmonic = m3

Augmented and Diminished Intervals - 3

original = ____ inversion = ____ enharmonic = ____

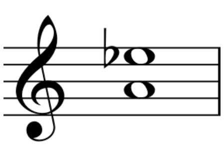
original = ____ inversion = ____ enharmonic = ____

original = ____ inversion = ____ enharmonic = ____

original = ____ inversion = ____ enharmonic = ____

original = ____ inversion = ____ enharmonic = ____

original = ____ inversion = ____ enharmonic = ____



original = ____ inversion = ____ enharmonic = ____

original = ____ inversion = ____ enharmonic =

Table of Contents

Diatonic Major Scales - 1

Name:________________________________ ID#________________

Using the equation immediately below (from p. 40 of the text),…

$M = \{p,(p+2),(p+4),(p+5),(p+7),(p+9),(p+11)\}$ (mod 12)

…and the given CPS number (p) representing the tonic pitch, give the numbers (from the CPS) for each pitch of the scale as well as the corresponding letter names.

example:

if p = 2 then M = { ____, ____, ____, ____, ____, ____, ____ } (mod 12)

M = { 2,4,6,7,9,11,1} (mod 12)

M = { D,E,F♯,G,A,B,C♯ }

p = 6 → M = { ____, ____, ____, ____, ____, ____, ____ } (mod 12)

M = { ____, ____, ____, ____, ____, ____, ____ }

p = 4 → M = { ____, ____, ____, ____, ____, ____, ____ } (mod 12)

M = { ____, ____, ____, ____, ____, ____, ____ }

p = 8 → M = { ____, ____, ____, ____, ____, ____, ____ } (mod 12)

M = { ____, ____, ____, ____, ____, ____, ____ }

p = 10 → M = { ____, ____, ____, ____, ____, ____, ____ } (mod 12)

M = { ____, ____, ____, ____, ____, ____, ____ }

Diatonic Major Scales - 2

p = 5 → M = { _____, _____, _____, _____, _____, _____, _____ } (mod 12)

M = { _____, _____, _____, _____, _____, _____, _____ }

p = 3 → M = { _____, _____, _____, _____, _____, _____, _____ } (mod 12)

M = { _____, _____, _____, _____, _____, _____, _____ }

p = 7 → M = { _____, _____, _____, _____, _____, _____, _____ } (mod 12)

M = { _____, _____, _____, _____, _____, _____, _____ }

p = 9 → M = { _____, _____, _____, _____, _____, _____, _____ } (mod 12)

M = { _____, _____, _____, _____, _____, _____, _____ }

p = 11 → M = { _____, _____, _____, _____, _____, _____, _____ } (mod 12)

M = { _____, _____, _____, _____, _____, _____, _____ }

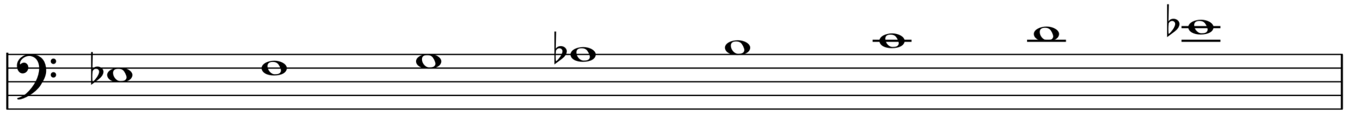
p = 1 → M = { _____, _____, _____, _____, _____, _____, _____ } (mod 12)

M = { _____, _____, _____, _____, _____, _____, _____ }

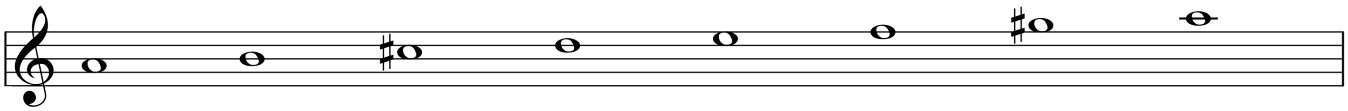
Diatonic Major Scales - 3

Name:_________________________________ ID#________________

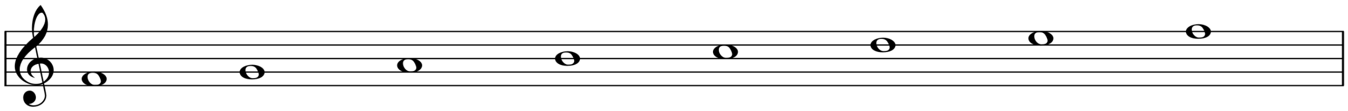
Referring to the whole-step/half-step pattern for major scales, identify which pitch in each of the following scales is incorrect and enter the correct pitch in the blank.



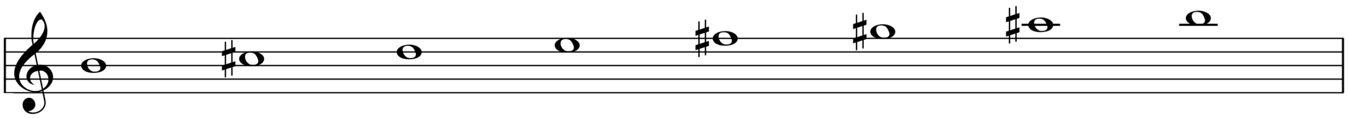
corrected pitch ____



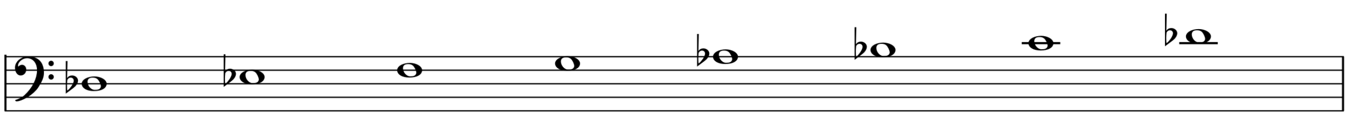
corrected pitch ____



corrected pitch ____

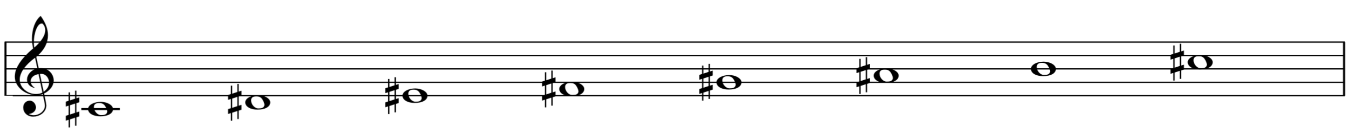


corrected pitch ____

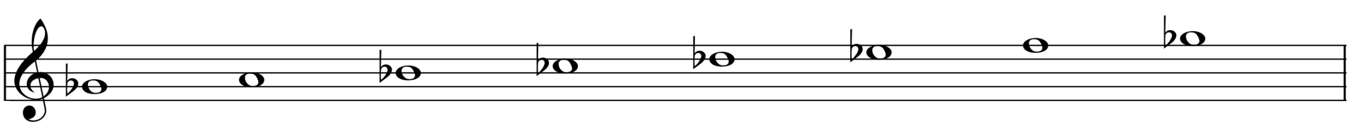


corrected pitch ____

corrected pitch ____



corrected pitch ____



corrected pitch ____

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Diatonic Minor Scales - 1

Name:________________________________ ID#___________

Using the equation immediately below (from p. 46 of the text),…

$m = \{p,(p+2),(p+3),(p+5),(p+7),(p+8),(p+10)\} \pmod{12}$

…and the given CPS number (p) representing the tonic pitch, give the numbers (from the CPS) for each pitch of the scale as well as the corresponding letter names.

example:

if p = 2 then m = { ____, ____, ____, ____, ____, ____, ____ } (mod 12)

m = { 2,4,5,7,9,10,0} (mod 12)

m = { D,E,F,G,A,B♭,C}

p = 6 → m = { ____, ____, ____, ____, ____, ____, ____ } (mod 12)

m = { ____, ____, ____, ____, ____, ____, ____ }

p = 4 → m = { ____, ____, ____, ____, ____, ____, ____ } (mod 12)

m = { ____, ____, ____, ____, ____, ____, ____ }

p = 8 → m = { ____, ____, ____, ____, ____, ____, ____ } (mod 12)

m = { ____, ____, ____, ____, ____, ____, ____ }

p = 10 → m = { ____, ____, ____, ____, ____, ____, ____ } (mod 12)

m = { ____, ____, ____, ____, ____, ____, ____ }

p = 5 → m = { ____, ____, ____, ____, ____, ____, ____ } (mod 12)

m = { ____, ____, ____, ____, ____, ____, ____ }

Diatonic Minor Scales - 2

p = 3 → m = { _____, _____, _____, _____, _____, _____, _____ } (mod 12)

m = { _____, _____, _____, _____, _____, _____, _____ }

p = 7 → m = { _____, _____, _____, _____, _____, _____, _____ } (mod 12)

m = { _____, _____, _____, _____, _____, _____, _____ }

p = 9 → m = { _____, _____, _____, _____, _____, _____, _____ } (mod 12)

m = { _____, _____, _____, _____, _____, _____, _____ }

p = 11 → m = { _____, _____, _____, _____, _____, _____, _____ } (mod 12)

m = { _____, _____, _____, _____, _____, _____, _____ }

p = 1 → m = { _____, _____, _____, _____, _____, _____, _____ } (mod 12)

m = { _____, _____, _____, _____, _____, _____, _____ }

Diatonic Minor Scales - 3

Name:________________________________ ID#___________

Using the scales noted above, identify and give the altered pitches that will be needed to create the harmonic minor and melodic minor versions.

example:

p = 2 pitch change for harmonic = C♯

pitch changes for ascending melodic = B♮, C♯

p = 4 pitch change for harmonic = ____
pitch changes for ascending melodic = ____, ____

p = 5 pitch change for harmonic = ____
pitch changes for ascending melodic = ____, ____

p = 7 pitch change for harmonic = ____
pitch changes for ascending melodic = ____, ____

p = 9 pitch change for harmonic = ____
pitch changes for ascending melodic = ____, ____

p = 10 pitch change for harmonic = ____
pitch changes for ascending melodic = ____, ____

p = 11 pitch change for harmonic = ____
pitch changes for ascending melodic = ____, ____

p = 8 pitch change for harmonic = ____
pitch changes for ascending melodic = ____, ____

p = 1 pitch change for harmonic = ____
pitch changes for ascending melodic = ____, ____

INTENTIONALLY BLANK

Diatonic Minor Scales - 4

Name:_________________________________ ID#____________

Complete each of the following tables, constructing the minor scales indicated by **p** as the tonic.

In the top row, give the CPS numbers for the PURE minor scale using this equation:
{p,(p+2),(p+3),(p+5),(p+7),(p+8),(p+10)} (MOD 12)

- In the second row, give the letter names (and accidentals) for the PURE minor scale.
- In the third row, give the letter names (and accidentals) for the HARMONIC version of the scale.
- In the fourth row, give the letter names (and accidentals) for the *ascending MELODIC version.

p = 9 →	m =									(MOD 12)
Pure										
Harmonic										
Melodic *										
		$\hat{1}$	$\hat{2}$	$\hat{3}$	$\hat{4}$	$\hat{5}$	$\hat{6}$	$\hat{7}$	$\hat{1}$	

p = 2 →	m =									(MOD 12)
Pure										
Harmonic										
Melodic *										
		$\hat{1}$	$\hat{2}$	$\hat{3}$	$\hat{4}$	$\hat{5}$	$\hat{6}$	$\hat{7}$	$\hat{1}$	

p = 7 →	m =									(MOD 12)
Pure										
Harmonic										
Melodic *										
		$\hat{1}$	$\hat{2}$	$\hat{3}$	$\hat{4}$	$\hat{5}$	$\hat{6}$	$\hat{7}$	$\hat{1}$	

Diatonic Minor Scales - 5

p = 0 →	m =									(MOD 12)
Pure										
Harmonic										
Melodic *										
		$\hat{1}$	$\hat{2}$	$\hat{3}$	$\hat{4}$	$\hat{5}$	$\hat{6}$	$\hat{7}$	$\hat{1}$	

p = 5 →	m =									(MOD 12)
Pure										
Harmonic										
Melodic *										
		$\hat{1}$	$\hat{2}$	$\hat{3}$	$\hat{4}$	$\hat{5}$	$\hat{6}$	$\hat{7}$	$\hat{1}$	

p = 10 →	m =									(MOD 12)
Pure										
Harmonic										
Melodic *										
		$\hat{1}$	$\hat{2}$	$\hat{3}$	$\hat{4}$	$\hat{5}$	$\hat{6}$	$\hat{7}$	$\hat{1}$	

p = 3 →	m =									(MOD 12)
Pure										
Harmonic										
Melodic *										
		$\hat{1}$	$\hat{2}$	$\hat{3}$	$\hat{4}$	$\hat{5}$	$\hat{6}$	$\hat{7}$	$\hat{1}$	

Diatonic Minor Scales - 6

p = 8 →	m =									(MOD 12)
Pure										
Harmonic										
Melodic *										
		$\hat{1}$	$\hat{2}$	$\hat{3}$	$\hat{4}$	$\hat{5}$	$\hat{6}$	$\hat{7}$	$\hat{1}$	

p = 1 →	m =									(MOD 12)
Pure										
Harmonic										
Melodic *										
		$\hat{1}$	$\hat{2}$	$\hat{3}$	$\hat{4}$	$\hat{5}$	$\hat{6}$	$\hat{7}$	$\hat{1}$	

p = 6 →	m =									(MOD 12)
Pure										
Harmonic										
Melodic *										
		$\hat{1}$	$\hat{2}$	$\hat{3}$	$\hat{4}$	$\hat{5}$	$\hat{6}$	$\hat{7}$	$\hat{1}$	

p = 11 →	m =									(MOD 12)
Pure										
Harmonic										
Melodic *										
		$\hat{1}$	$\hat{2}$	$\hat{3}$	$\hat{4}$	$\hat{5}$	$\hat{6}$	$\hat{7}$	$\hat{1}$	

Diatonic Minor Scales - 7

p = 4 →	m =									(MOD 12)
Pure										
Harmonic										
Melodic *										
		$\hat{1}$	$\hat{2}$	$\hat{3}$	$\hat{4}$	$\hat{5}$	$\hat{6}$	$\hat{7}$	$\hat{1}$	

Major and Minor Keys - 1

Name:________________________________ ID#___________

Referring to the charts on page 48 of the text, give the scale degree <u>name</u> for each of the following.

example:

If F = $\hat{1}$ then C = _______________ answer: Dominant

If G = $\hat{1}$ then B = _______________ If B♭ = $\hat{2}$ then D♭ = _______________

If D = $\hat{3}$ then G = _______________ If F = $\hat{4}$ then C = _______________

If A♭ = $\hat{5}$ then E♭ = _______________ If C = $\hat{6}$ then D = _______________

If E = $\hat{7}$ then B♭ = _______________ If G♯ = $\hat{3}$ then C♯ = _______________

If B = $\hat{4}$ then F♯ = _______________ If D♭ = $\hat{5}$ then F = _______________

If F♯ = $\hat{3}$ then E = _______________ If A = $\hat{4}$ then D♯ = _______________

If C♯ = $\hat{2}$ then G♯ = _______________ If E♭ = $\hat{5}$ then A♭ = _______________

If G♭ = $\hat{4}$ then E♭ = _______________ If D♭ = $\hat{2}$ then F♭ = _______________

Major and Minor Keys - 2

Arrange the keys in order by taking the <u>numbers</u> from the list of keys on the left and putting them in the correct order (top to bottom) in the blanks on the right.

1 B♭ major ____

2 C major ____

3 A♭ major ____

4 C♭ major ____

5 E♭ major ____

6 G♭ major ____

7 F major ____

8 D♭ major ____

Arrange the keys in order by taking the numbers from the list of keys on the left and putting them in the correct order (top to bottom) in the blanks on the right.

1 G major ____

2 C major ____

3 A major ____

4 C♯ major ____

5 E major ____

6 B major ____

7 F♯ major ____

8 D major ____

Major and Minor Keys - 3

Name:________________________________ ID#___________

Identify each of the following key signatures and give its relative minor key.

Major key _____ Relative minor key _____

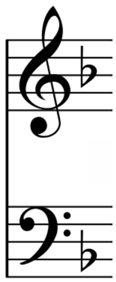
Major key _____ Relative minor key _____

Major key _____ Relative minor key _____

Major key _____ Relative minor key _____

Major and Minor Keys - 4

Major key _____ Relative minor key _____



Major key _____ Relative minor key _____

Major key _____ Relative minor key _____

Major key _____ Relative minor key _____

Major key _____ Relative minor key _____

Major and Minor Keys - 5

Major key ______ Relative minor key ______

Major key ______ Relative minor key ______

Major key ______ Relative minor key ______

Major key ______ Relative minor key ______

Major key ______ Relative minor key ______

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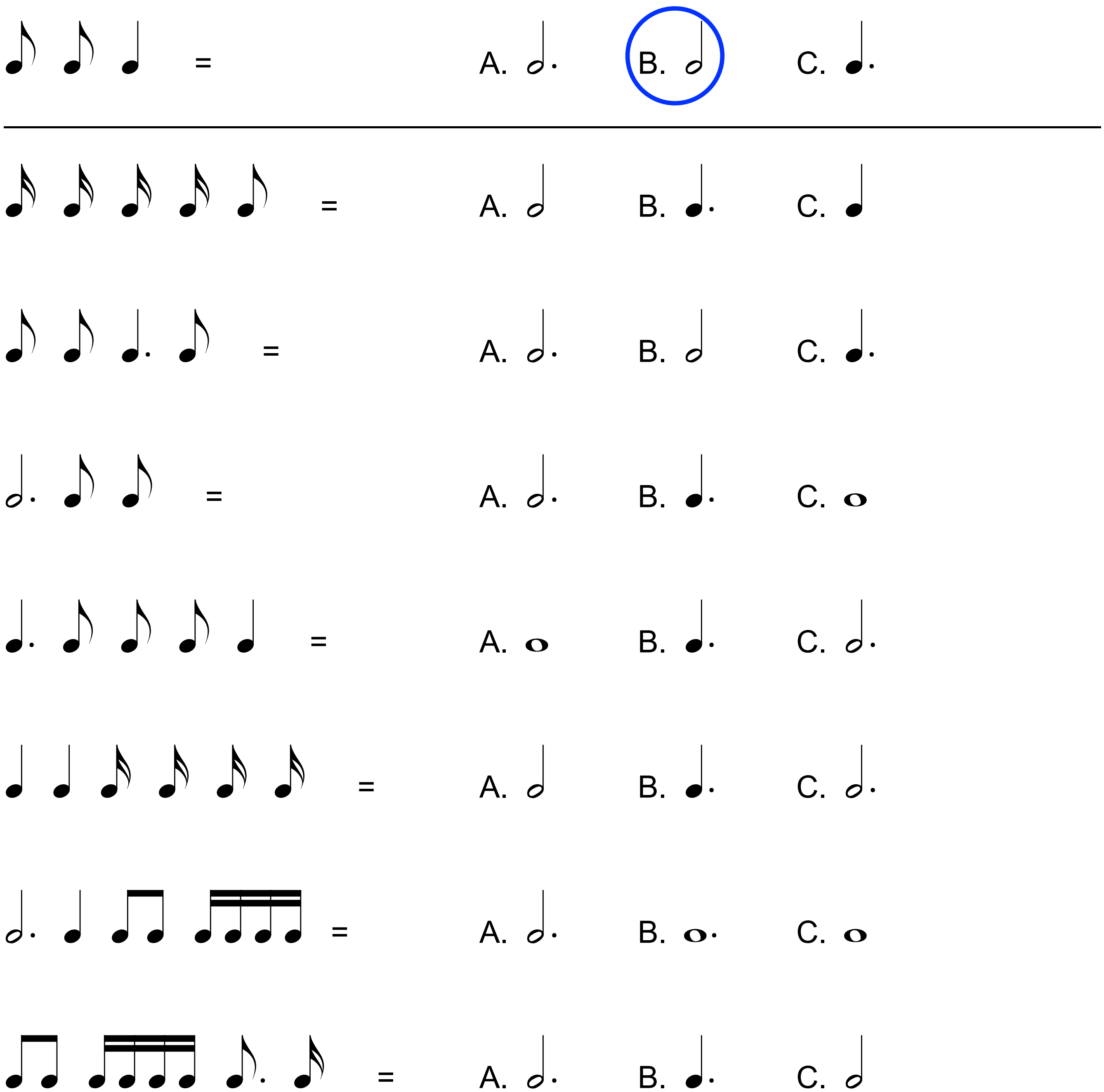
Note Values and Meter - 1

Name:________________________________ ID#___________

Add the note values on the left together then choose the correct equivalent value on the right. (Circle the correct answer.)

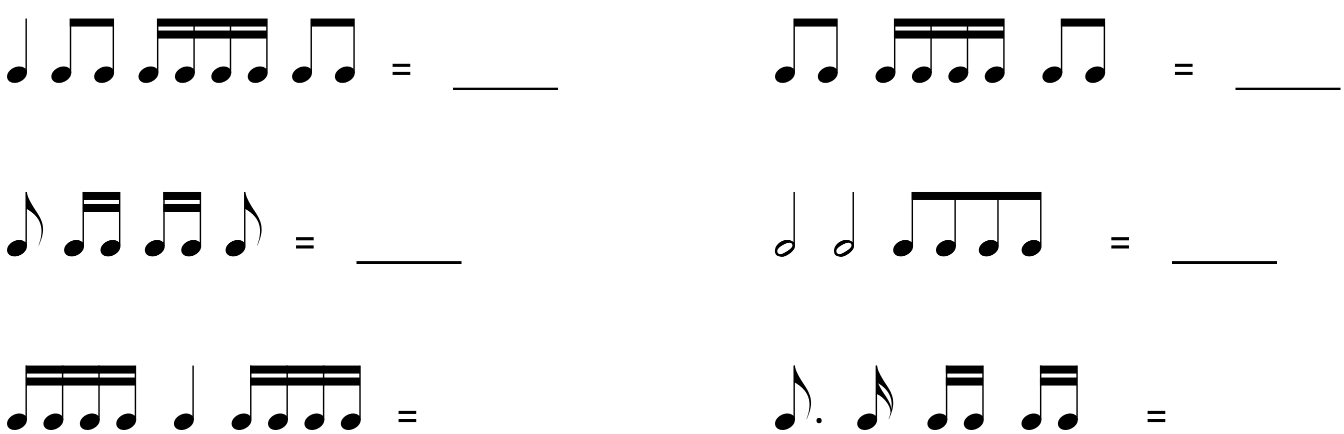
example:



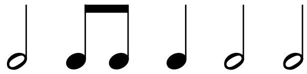
Note Values and Meter - 2

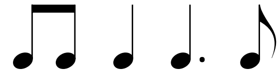
For each of the following note groupings, based on note values and beamed groups, give the best possible meter. (Designate the meter like this: 3/4, 4/2, etc.)

example:  = ______ answer: 4/4

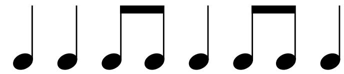


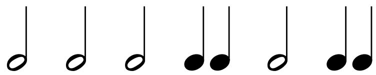
Note Values and Meter - 3

 = _____

 = _____

 = _____

 = _____

 = _____

 = _____

INTENTIONALLY BLANK

Compound and Asymmetrical Meter - 1

Name:_________________________________ ID#____________

For each of the compound meters given:
- identify and circle the note value that gets one beat
- give the mathematical designation for that note value
- give the number of beats in one measure

example:

$\frac{6}{8}$	A ♪ B ♩. C 𝅗𝅥.	$^3/_8$	2

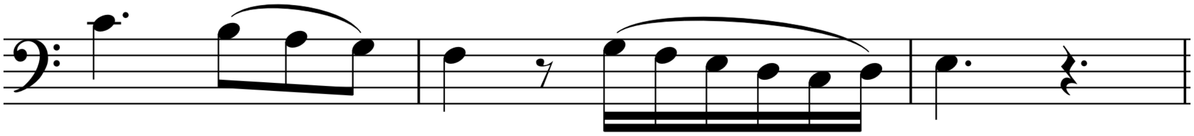
$\frac{9}{8}$	A ♪ B ♩. C 𝅗𝅥.		
$\frac{6}{4}$	A ♪. B ♩ C 𝅗𝅥.		
$\frac{12}{16}$	A ♪. B ♩. C 𝅗𝅥		
$\frac{9}{4}$	A ♩ B 𝅗𝅥. C 𝅝.		
$\frac{6}{2}$	A ♩ B 𝅗𝅥 C 𝅝.		
$\frac{6}{16}$	A ♪. B ♩ C 𝅝.		
$\frac{12}{8}$	A ♪. B ♩. C 𝅗𝅥		
$\frac{9}{16}$	A ♪. B ♩ C 𝅝.		
$\frac{12}{4}$	A ♩. B 𝅗𝅥. C 𝅝.		
$\frac{9}{2}$	A ♩. B 𝅗𝅥 C 𝅝.		

Table of Contents

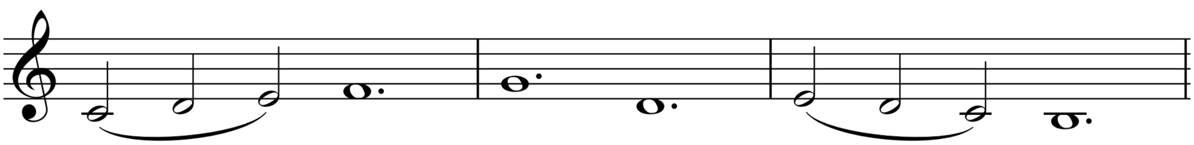
Compound and Asymmetrical Meter - 2

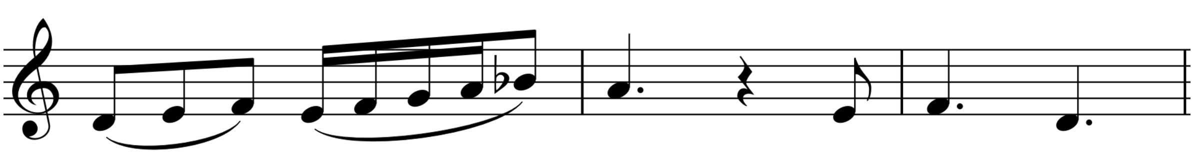
For each of the musical excerpts below, identify the compound meter.

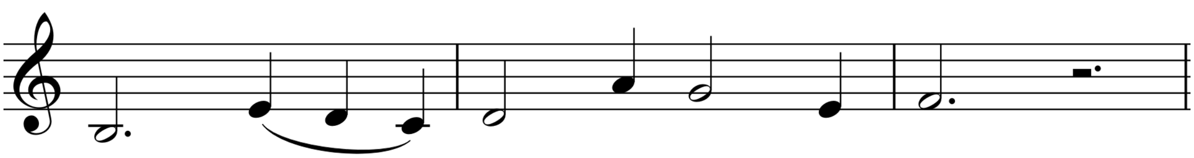
example:

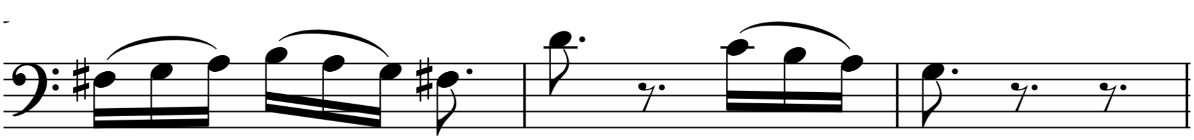


6
8

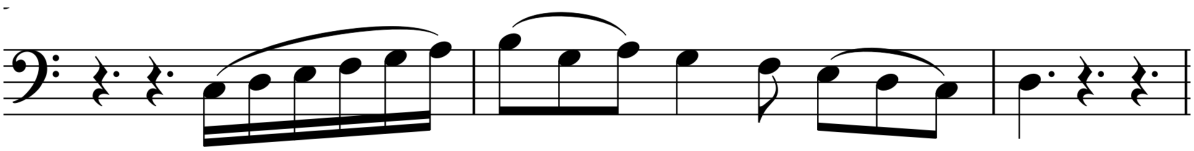


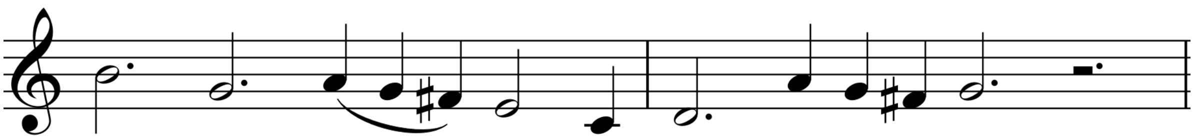


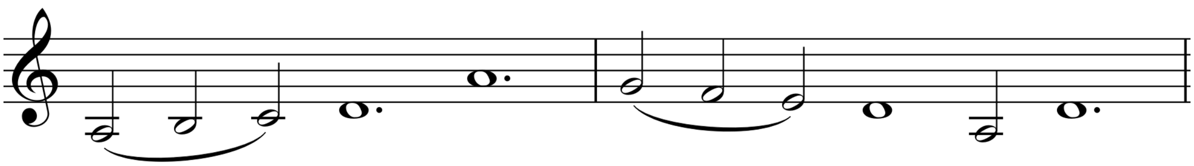




Compound and Asymmetrical Meter - 3

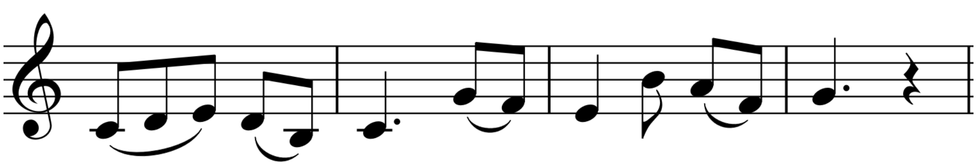




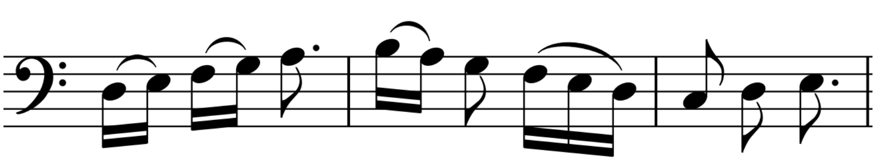


For each of the musical excerpts given:
- identify and give the asymmetrical meter
- give the mathematical equation for the subdivisions of the beats
- give the number of beats in one measure

example:

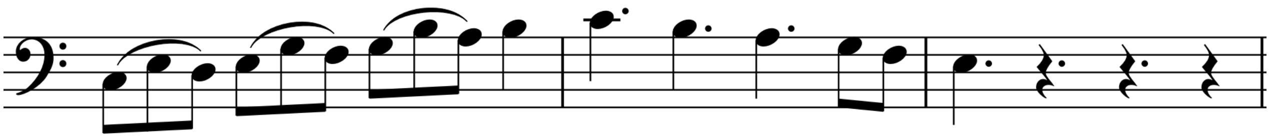


meter	subdivisions	number of beats
$\frac{5}{8}$	3 + 2	2



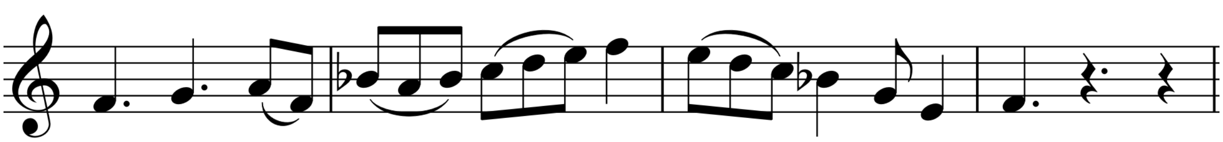
meter	subdivisions	number of beats

Compound and Asymmetrical Meter - 4



meter	subdivisions	number of beats

meter	subdivisions	number of beats



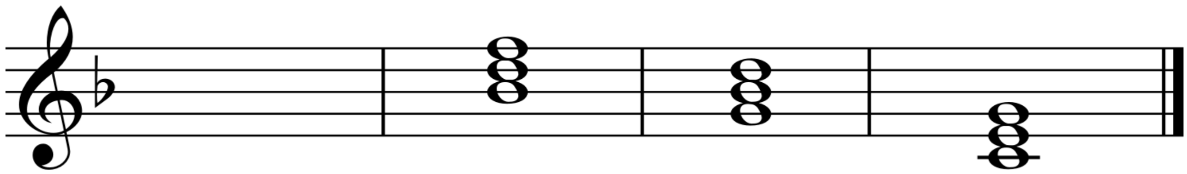
meter	subdivisions	number of beats

Triads - 1

Name:________________________________ ID#___________

For each of the keys noted below, write the triads on the staff for each of the Roman numerals given.

example:



F major IV ii V

D major I IV ii

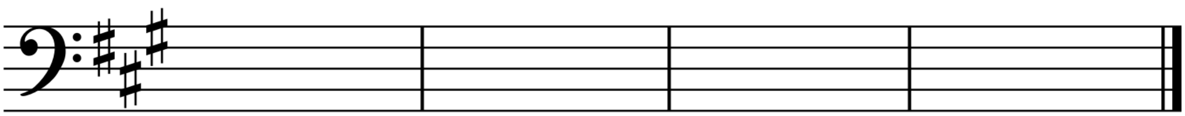
B minor i iv III

E♭ major ii V vi

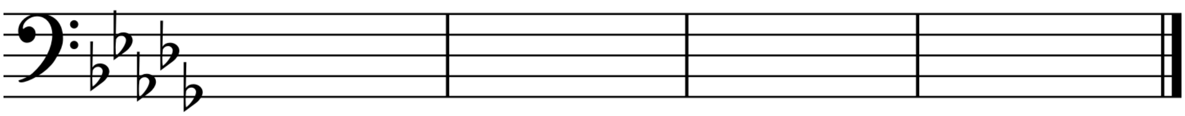
C minor III VI iv

Triads - 2

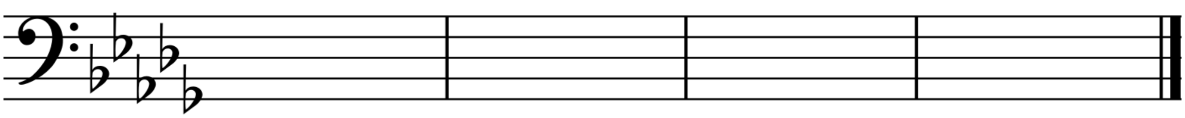
A major iii IV ii



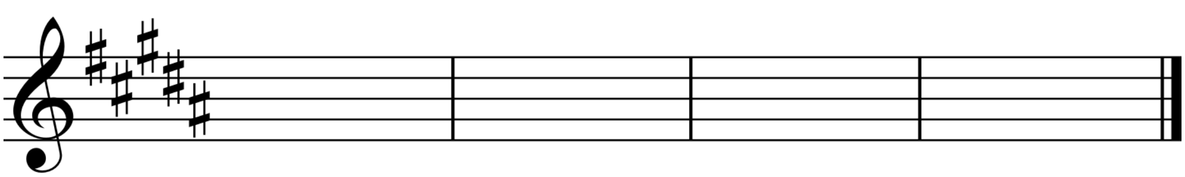
F♯ major i VI iv



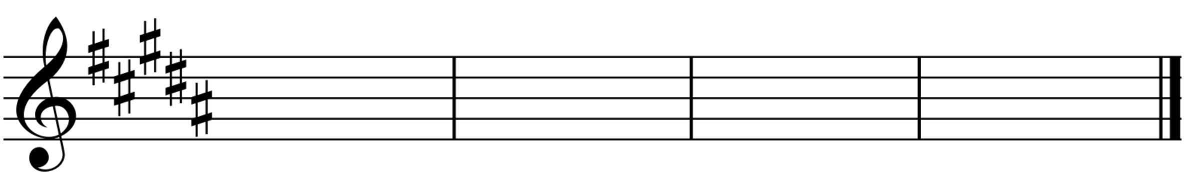
D♭ major ii vii° IV



B♭ minor III iv i



B major vi IV V



G♯ minor III i VI

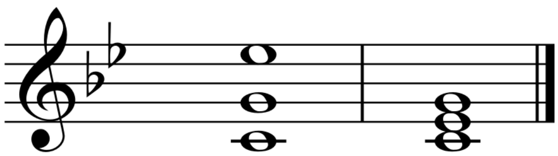
Triads - 3

Name:________________________________ ID#___________

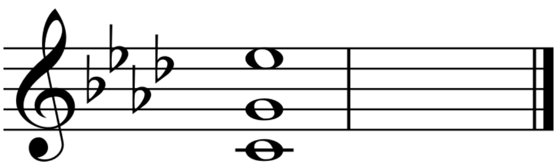
For the next exercise, use these Roman numerals for major and minor keys.

Major: I ii iii IV V vi vii° **Minor**: i ii° III iv v VI VII

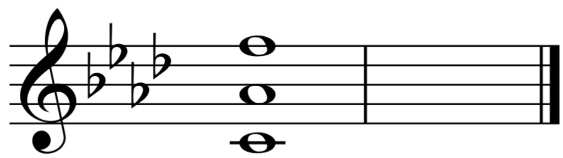
For each of the chords given in open voicing, write the triad in root position in closed voicing and give the Roman numeral.

example: 

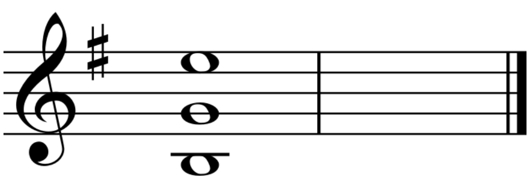
B♭ major ii



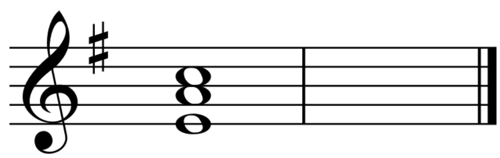
A♭ major ____



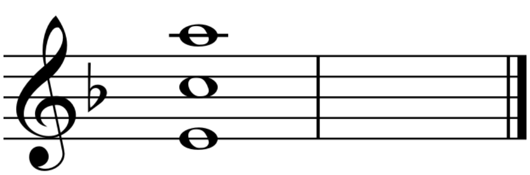
F minor ____



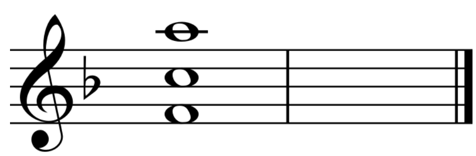
G major ____



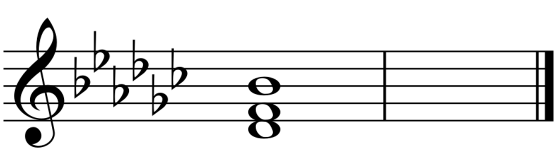
E minor ____



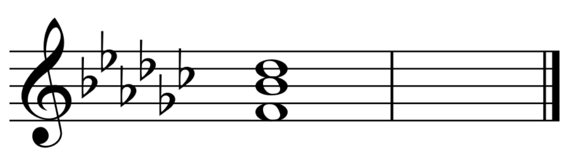
F major ____



D minor ____



G♭ major ____



E♭ minor ____

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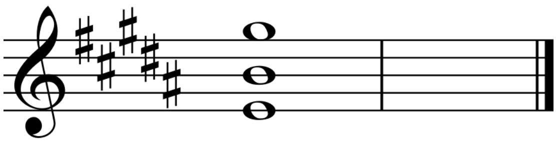
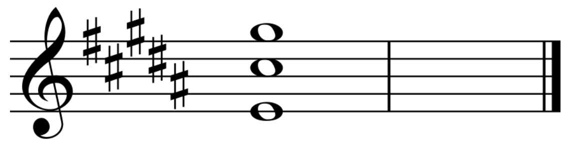
Triads - 4



B major ____



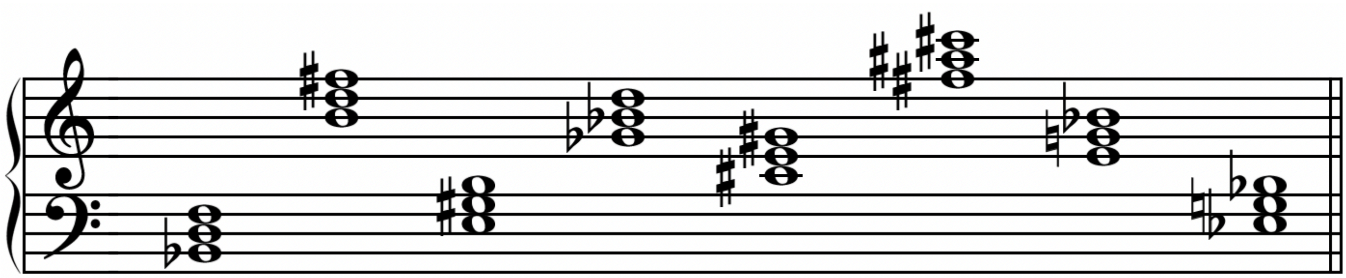
G♯ minor ____

Table of Contents

Chord Quality - 1

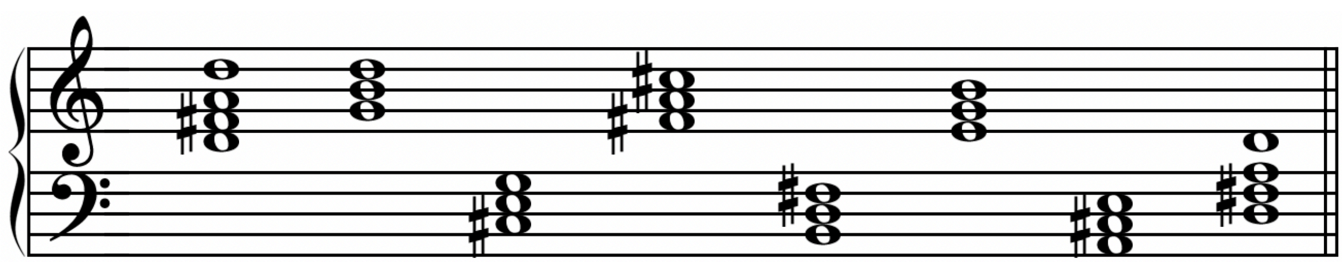
Name:_________________________________ ID#____________

For each of the following triads, identify the quality. Write the appropriate label under each chord. (Each triad is independent, not based on a specific key.)



Identify the key of each selection below based on the accidentals. **<u>An upper case letter will signify a major key and lower case will be minor</u>**.

Label each of the triads with the appropriate Roman numeral. (Hint: The root of the first and last chord of each selection will be the tonic pitch of the key.) (Another hint: not every triad will be in root or closed position.) (Last hint: Watch for V in minor keys.)

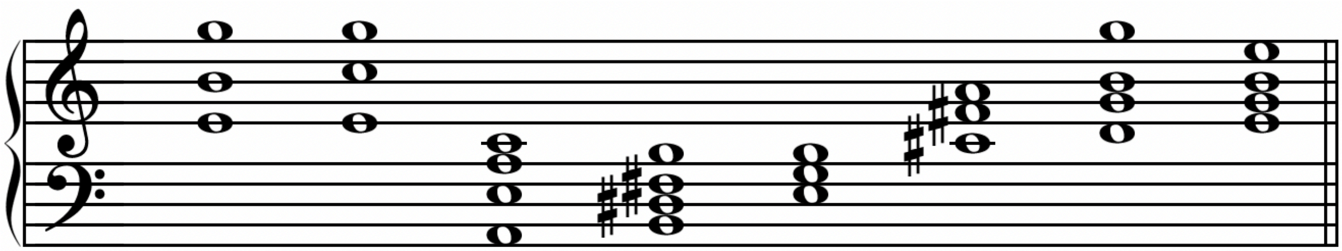


Key _____

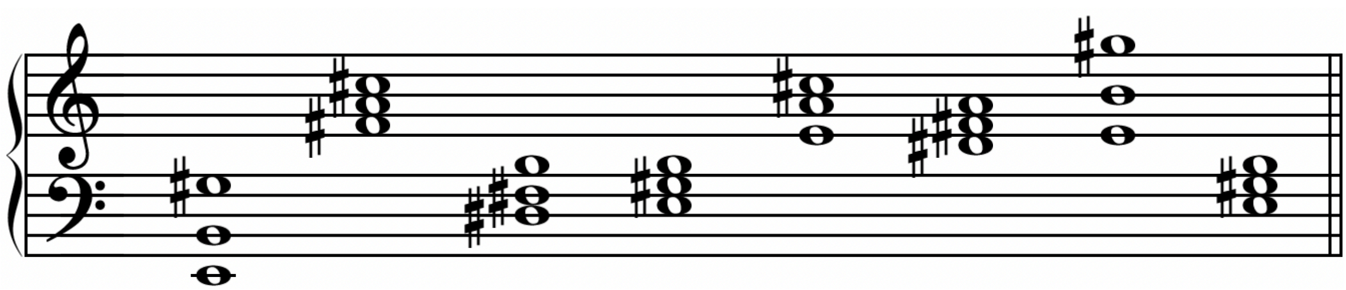
Table of Contents

Chord Quality - 2

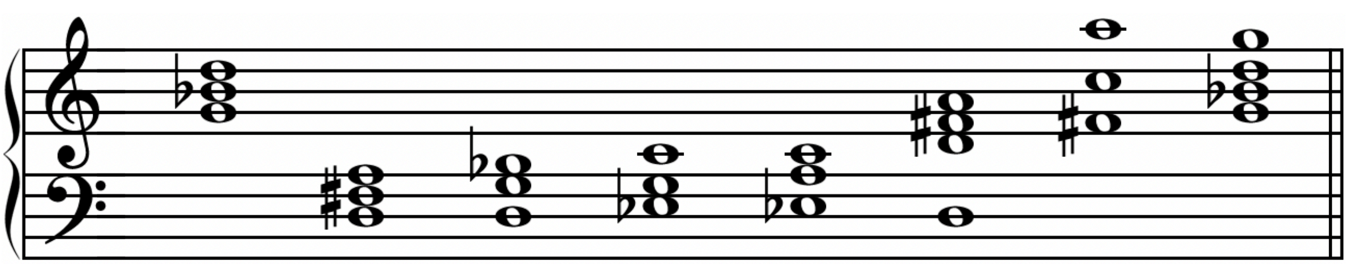
Key ____



Key ____



Key ____



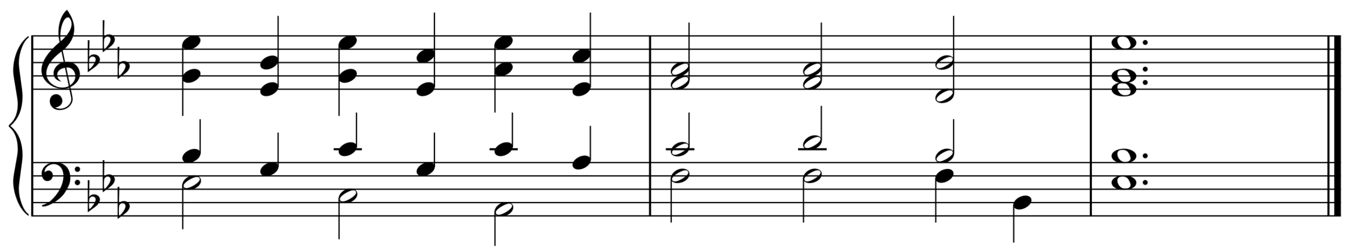
Key ____

Chord Quality - 3

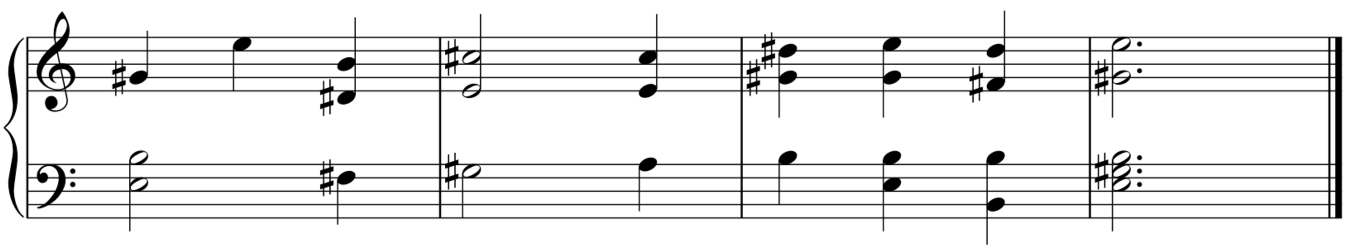
Name:_________________________________ ID#___________

For each of the following selections, identify the key and the triads. Label the triads with the appropriate Roman numeral. (Note: some selections have the key signature and some do not. If there is not a key signature, you must identify the key by the music itself.)

Chord Quality - 4



Chord Quality - 5



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7th Chords - 1

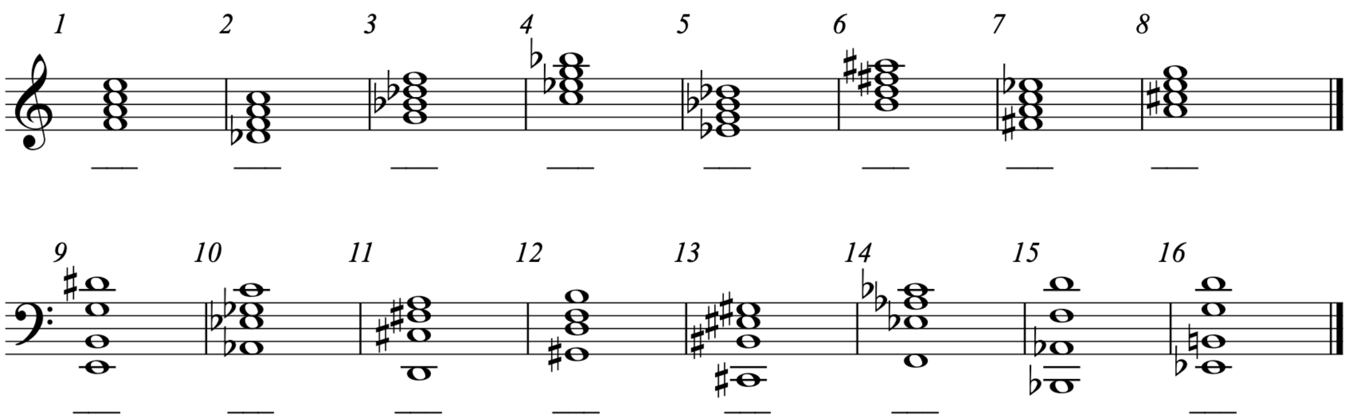
Name:________________________________ ID#___________

Complete this table with the appropriate interval qualities.

Quality of the 7th chord	Distance from R up to 3rd	Distance from 3rd up to 5th	Distance from 5th up 7th	Distance from 7th up to R
MM				
Mm				
mM				
mm				
dm				
dd				
AM				

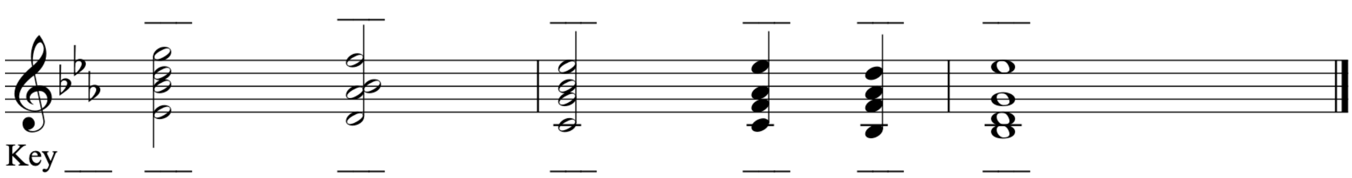
Identify the quality and label each of the following 7th chords.

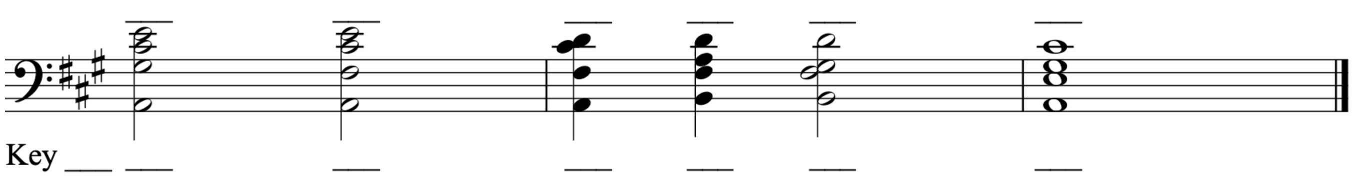
These are the labels you should use: MM Mm mm mM dm dd AM

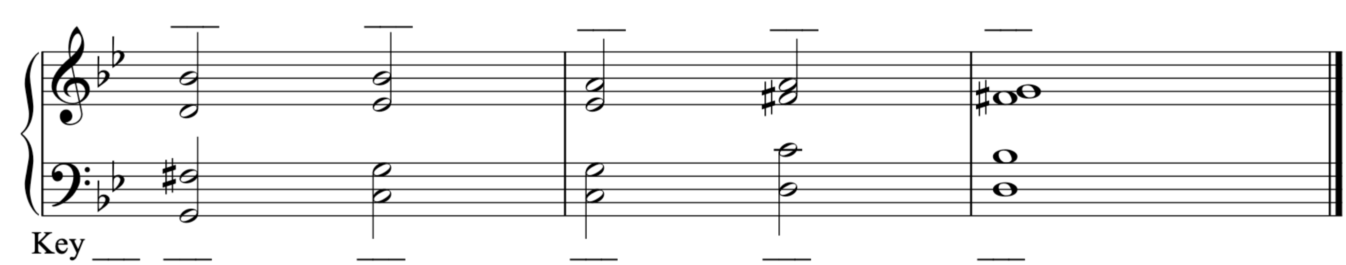


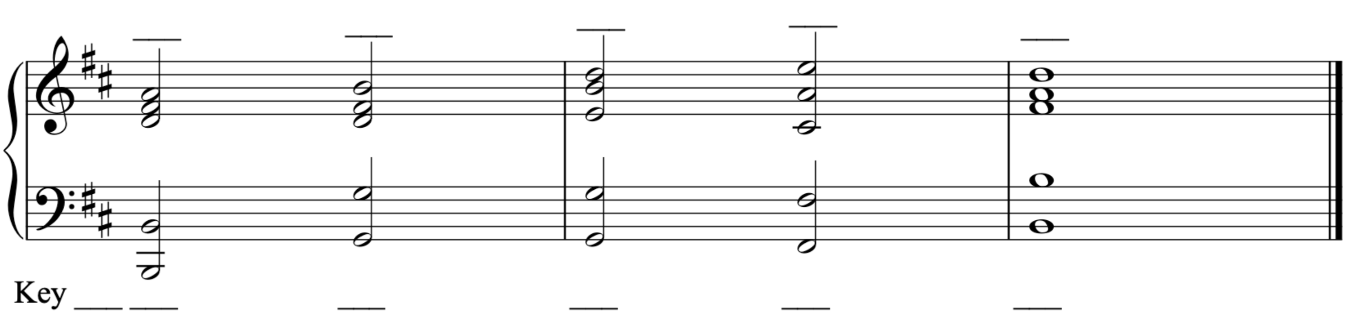
7th Chords - 2

Identify each of the following <u>diatonic</u> 7th chords. In the blank above the chord write the quality. In the blanks below, give the key then write the appropriate Roman numerals.







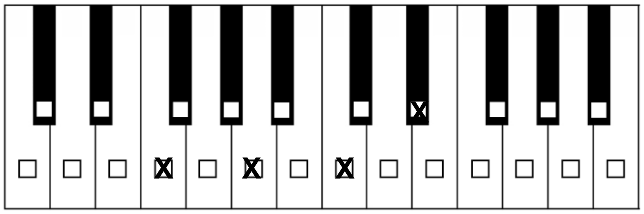


7th Chords - 3

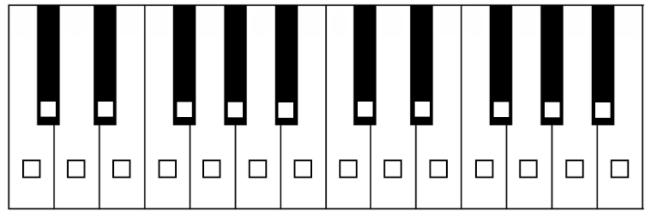
Name:_________________________________ ID#____________

For the 7th chords given below, indicate how they would appear on the keyboard. Put the chord in root position and place an X in the box of each key/pitch of the chord. The chords are indicated with the letter name of the root, then the quality of the 7th chord. The first one is completed as an example.

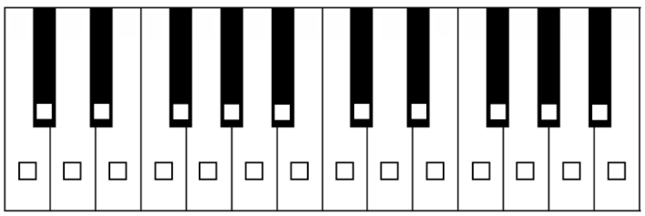
F - Mm



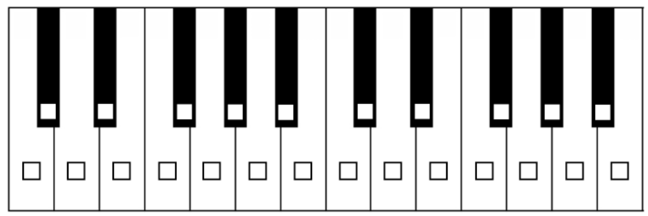
E♭ - mm



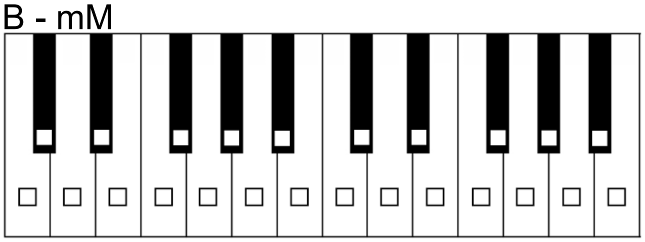
D - dm



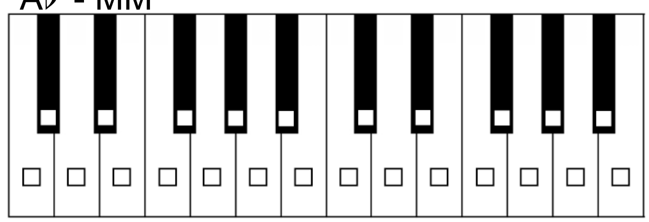
C♯ - dd



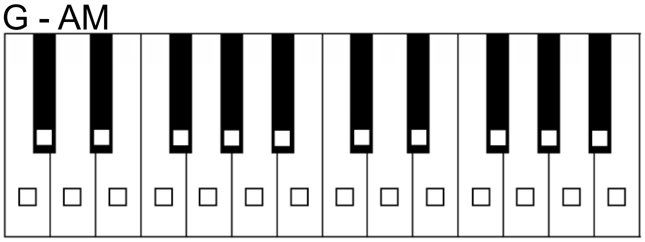
B - mM



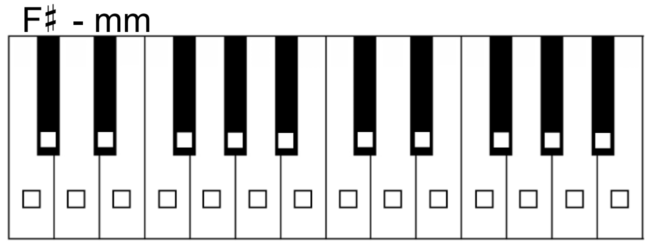
A♭ - MM



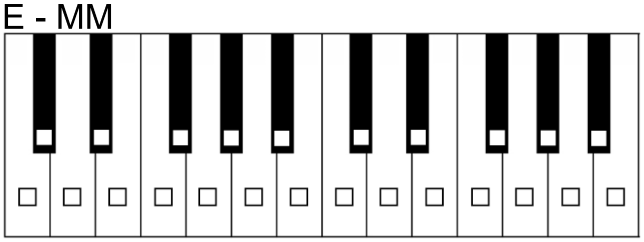
G - AM



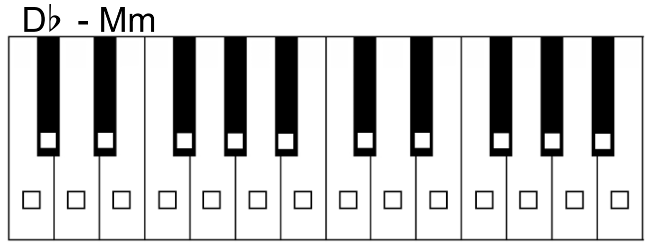
F♯ - mm



E - MM

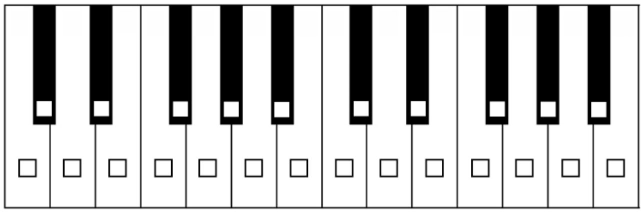


D♭ - Mm

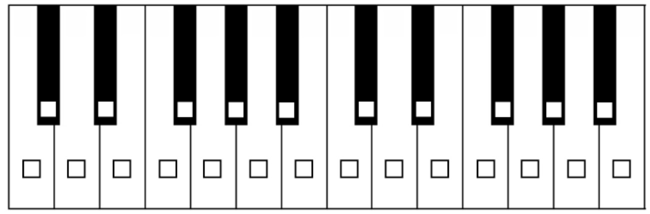


7th Chords - 4

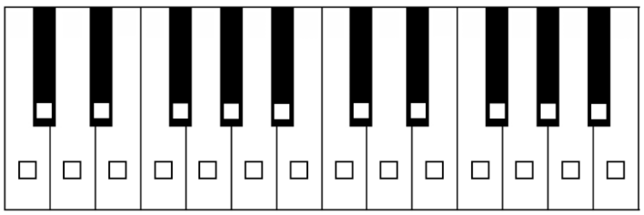
C - mM



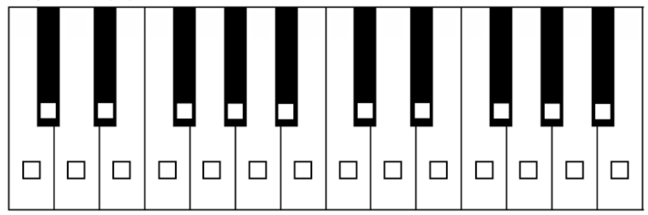
B♭ - dm



A - AM



G♯ - dd



Inversions and Figures Bass - 1

Name:______________________________ ID#__________

Complete the following table

Chord member in the bass	**R** (triad)	**3rd** (triad)	**5th** (triad)	**R** (7th chord)	**3rd** (7th chord)	**5th** (7th chord)	**7th** (7th chord)
Figured bass typically used							

How do you find the root of a triad if:

• the 3rd is in the bass ______________________________

• the 5th is in the bass ______________________________

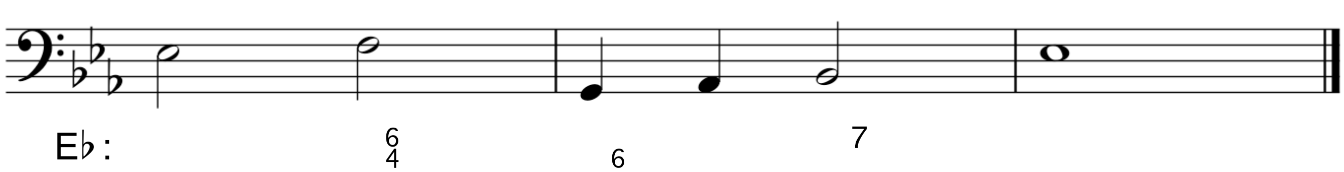
How do you find the root of a 7th chord if:

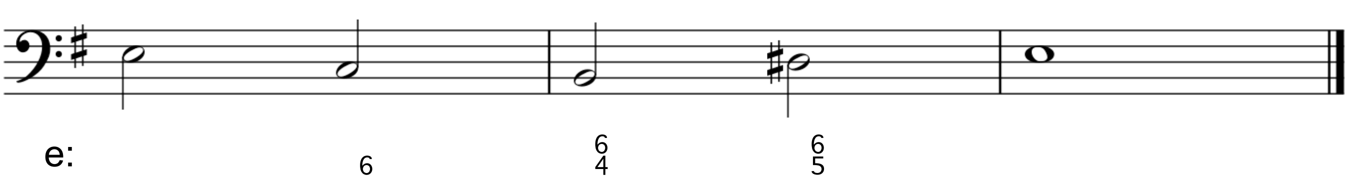
• the 3rd is in the bass ______________________________

• the 5th is in the bass ______________________________

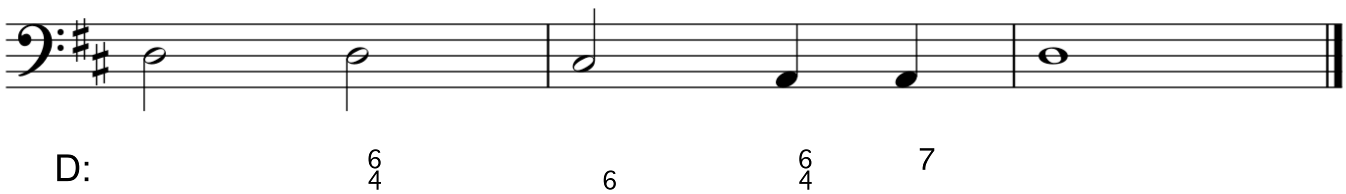
• the 7th is in the bass ______________________________

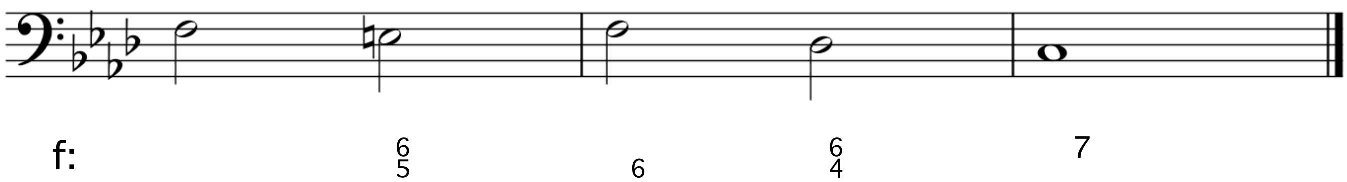
For each of the following bass lines with figured bass, notate the chord (in closed voicing) above the given bass note and give the appropriate Roman numeral for the chord.



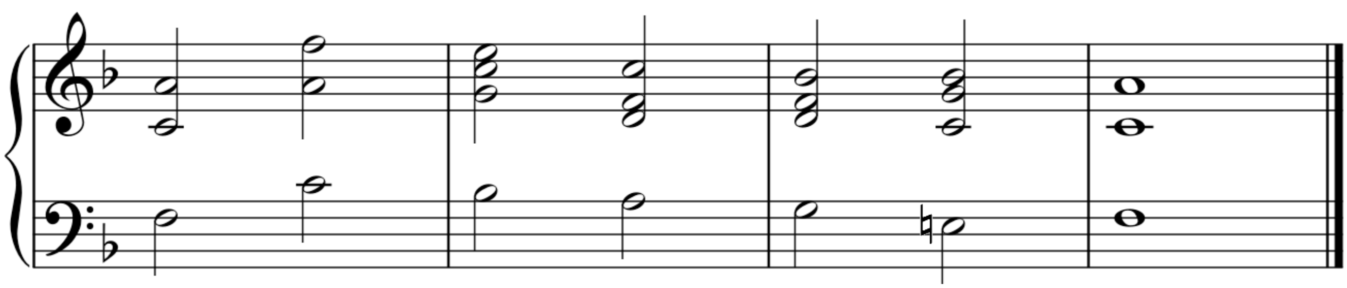


Inversions and Figures Bass - 2

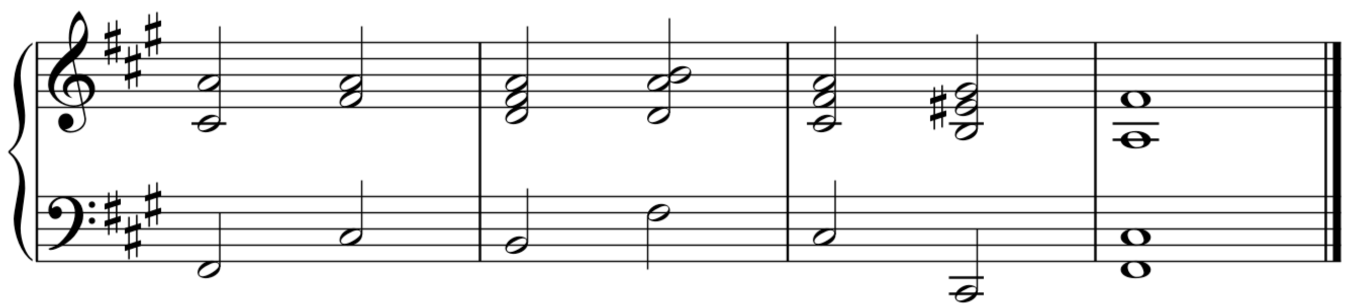


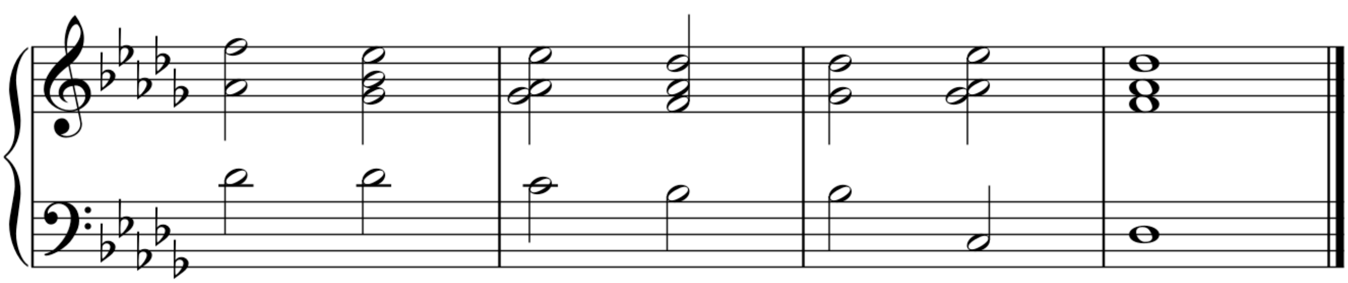


Give a complete harmonic analysis of the following selections. Include the key, Roman numerals, and figured bass.



Inversions and Figures Bass - 3





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7th Chords, Inversions, and Figures Bass - 1

Name:________________________________ ID#___________

Complete this table with the appropriate interval qualities.

Quality of the 7th chord	Distance from R up to 3rd	Distance from 3rd up to 5th	Distance from 5th up 7th	Distance from 7th up to R
MM				
Mm				
mM				
mm				
dm				
dd				
AM				

In the tables below, put the appropriate quality of the 7th chords in the blank boxes above the Roman numerals.

Major

Quality of the 7th chord							
Roman numeral	I^7	ii^7	iii^7	IV^7	V^7	vi^7	$\mathrm{vii}^{ø7}$

Pure Minor

Quality of the 7th chord							
Roman numeral	i^7	$\mathrm{ii}^{ø7}$	III^7	iv^7	v^7	VI^7	VII^7

Harmonic Minor

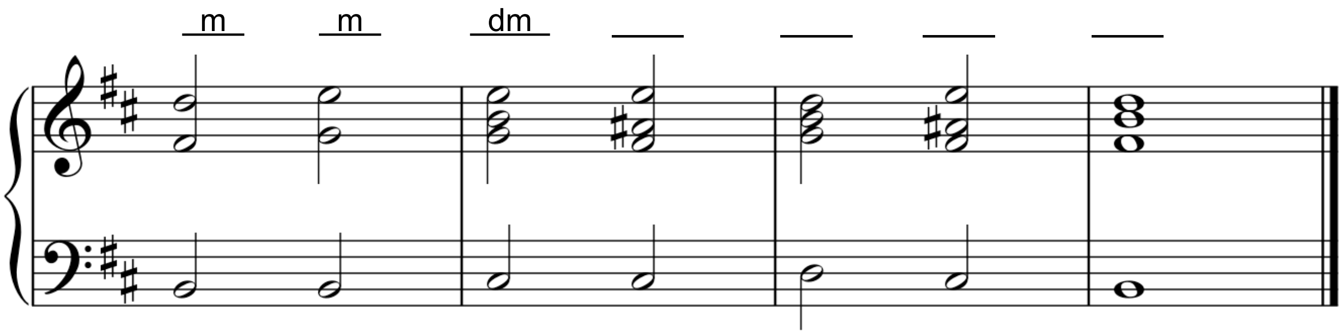
Quality of the 7th chord							
Roman numeral	i^7	$\mathrm{ii}^{ø7}$	III^7	iv^7	V^7	VI^7	vii^{o7}

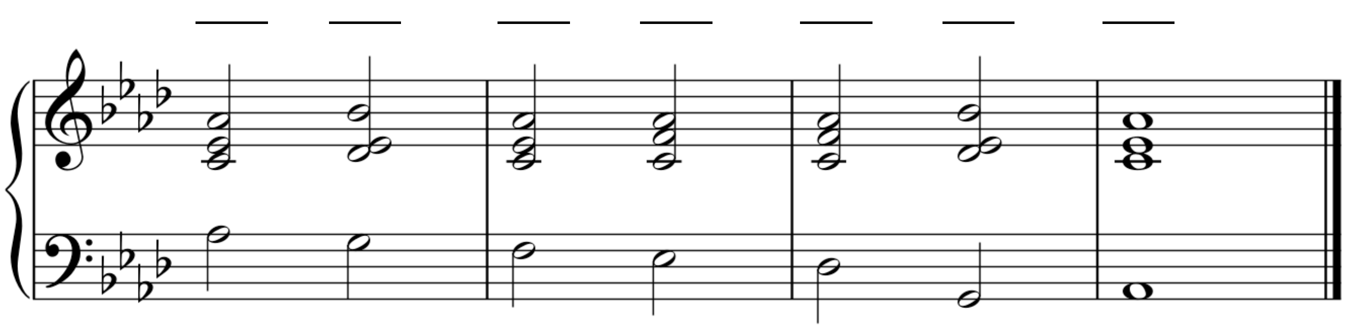
Ascending Melodic Minor

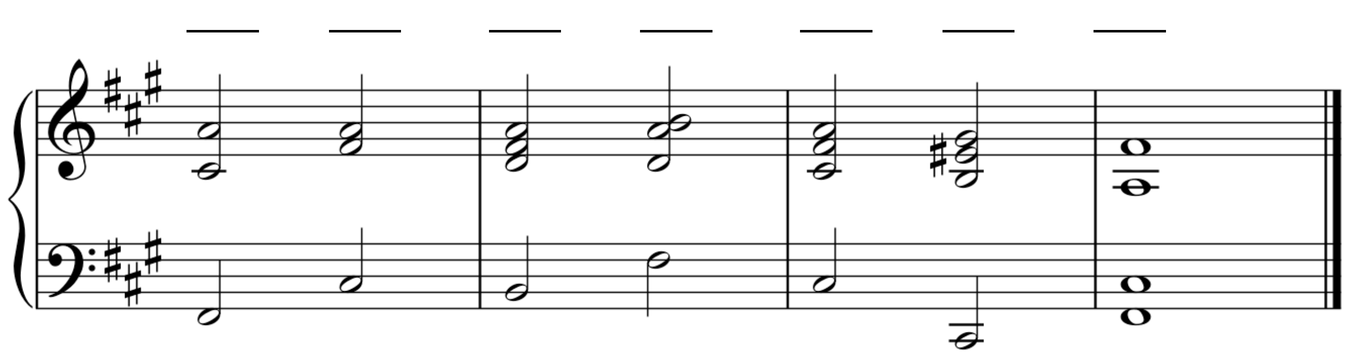
Quality of the 7th chord							
Roman numeral	i^7	$\mathrm{ii}^{ø7}$	III^7	iv^7	V^7	$\mathrm{vi}^{ø7}$	vii^{o7}

7th Chords, Inversions, and Figures Bass - 2

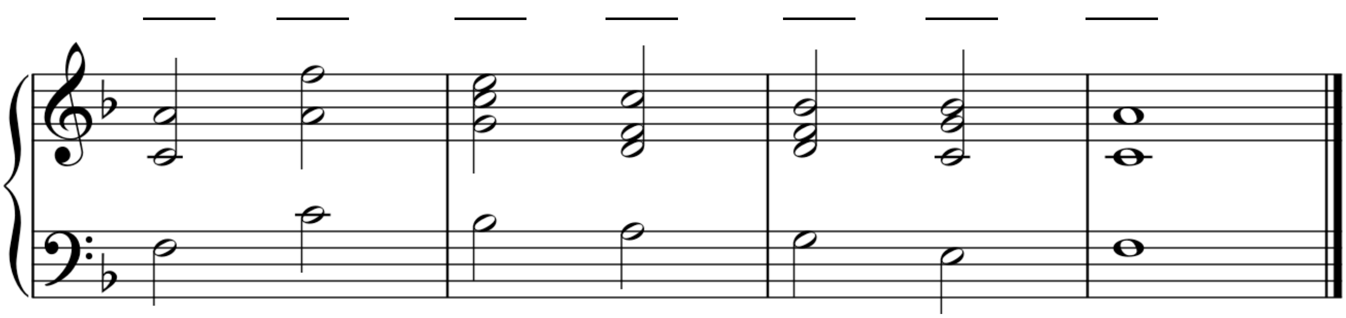
Give a complete harmonic analysis of the following selections. Include the key, Roman numerals, and figured bass. In the blanks above the chords write the chord quality. If it has no 7th, you will only need to put a one-letter label for the triad. If it has a 7th, it will need two letters to identify it. (See the example immediately below)

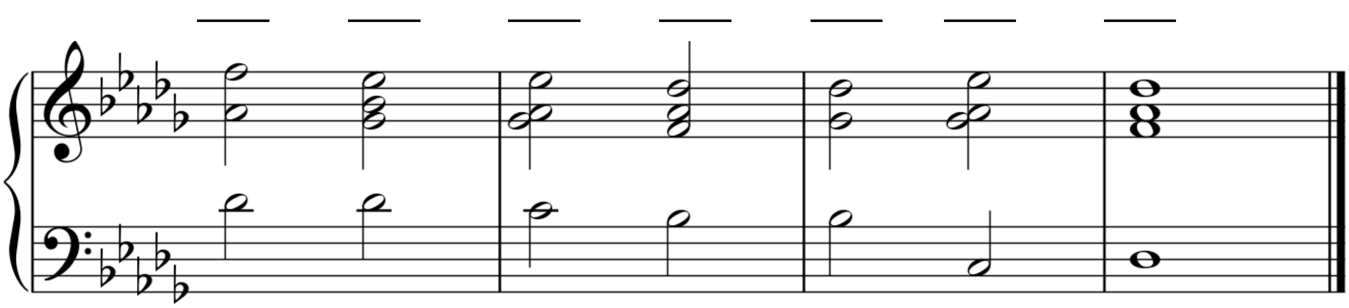






7th Chords, Inversions, and Figures Bass - 3





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Strong Root Movement - 1

Name:________________________________ ID#___________

Identify the root movements between each of the chords in the following progressions.
Use the following abbreviations: **D5** = down a 5th; **D3** = down a 3rd; **U2** = up a 2nd
If the root movement is one of the exceptions put **E**. If it's a weak movement put **W**.

i ____ VI$_{6}$ ____ iv$^{6}_{4}$ ____ vii^{o7} ____ III$^{6}_{4}$ ____ iv$^{6}_{4}$ ____ i

---

I ____ ii$^{4}_{2}$ ____ V$^{6}_{5}$ ____ vi ____ ii$^{6}_{4}$ ____ vii^{o} ____ I

I ____ iii$^{6}_{4}$ ____ IV$^{6}_{4}$ ____ ii$^{4}_{2}$ ____ iii$^{6}_{4}$ ____ vii^{ø7} ____ I

---

i ____ iv$^{6}_{4}$ ____ v$_{6}$ ____ iv$_{6}$ ____ i$^{6}_{4}$ ____ V^{7} ____ i

Strong Root Movement - 2

For each of the following progressions, put the root movements in the blanks above the staff.

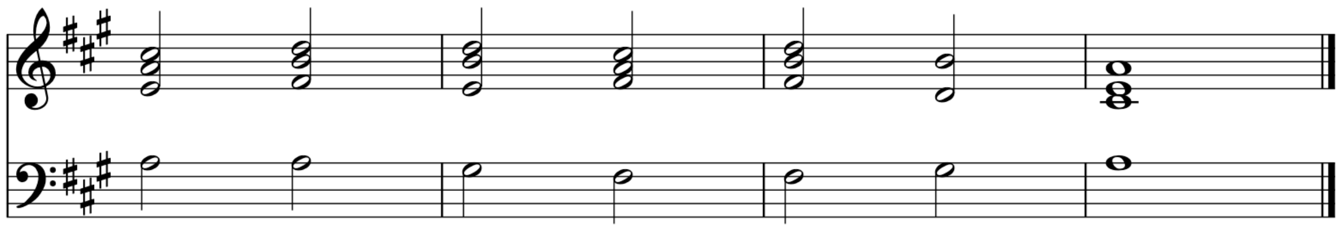
Underneath the staff put the key, then put the **scale degree** that is the root of each chord. Remember: Scale degrees have the circumflex above.

KEY

KEY

KEY

Strong Root Movement - 3



KEY

For each of the following tables, create a chord progression using **ONLY strong root movements (including the Exceptions)**. In the blanks, put the **scale degree** of the chord's root. In the blanks above put the type of movement it is (D5, D3, U2, E). Note: They begin and end on $\hat{1}$.

$\hat{1}$						$\hat{1}$

$\hat{1}$						$\hat{1}$

INTENTIONALLY BLANK

Cadences - 1

Name:________________________________ ID#___________

What are the names of the two phrases that comprise a full musical period (in order)?

___________________________ ___________________________

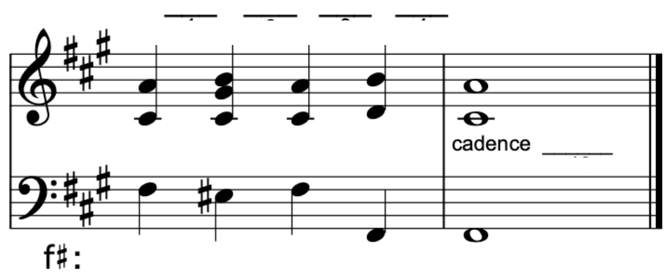
What cadence is typically found at the end of the first phrase? ___________________

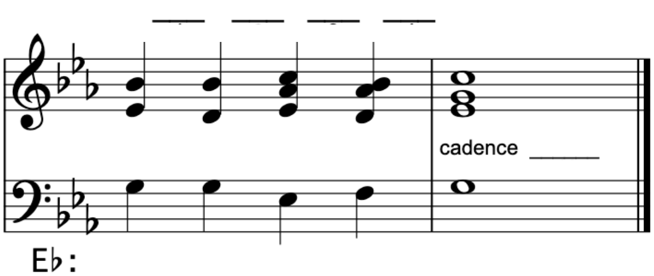
What cadence is typically found at the end of the second phrase? ________________

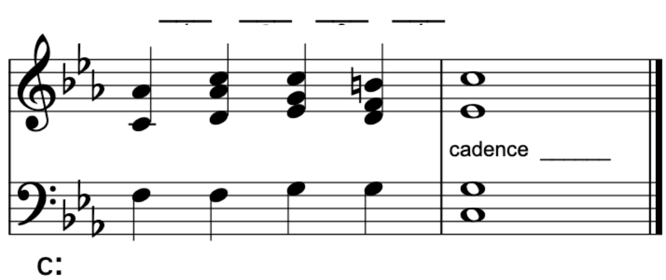
____

For the following musical excerpts:

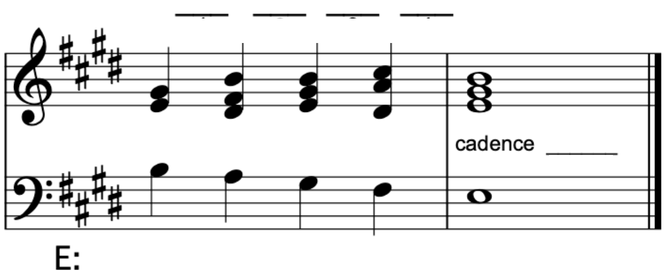
- give the Roman numerals and figured bass for each chord (below the staff)
- give the root movements in the blanks above the staff (D5, D3, U2, E, W)
- identify and label each cadence
  (with the abbreviations - PAC, IAC, HC, PC, DC)

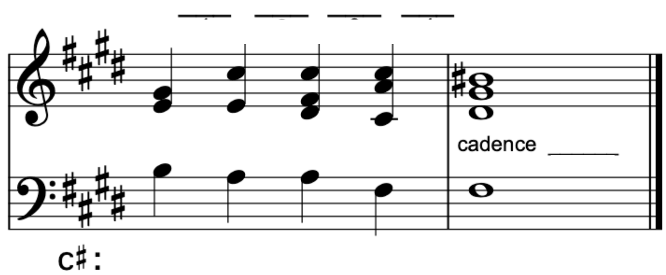


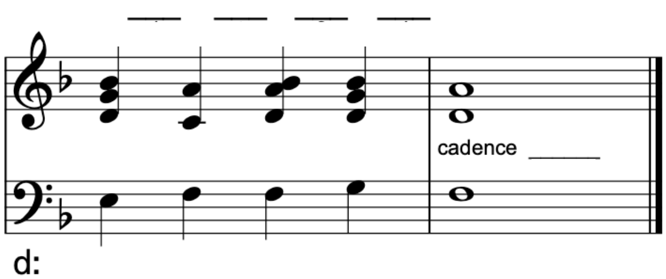


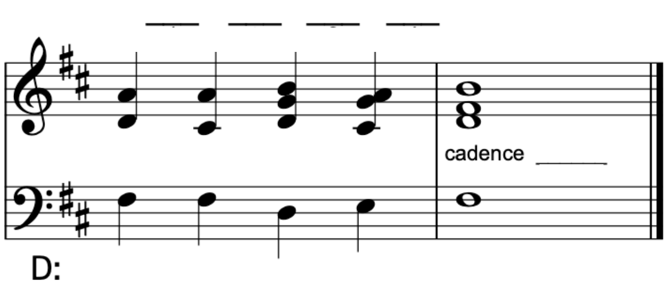


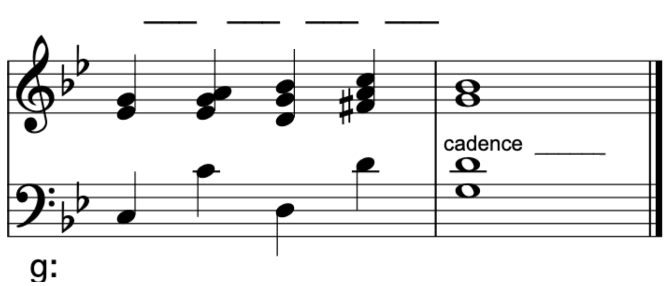
Cadences - 2











Voice-Leading - 1

Name:________________________________ ID#___________

Let's take our Voice-Leading guidelines, break them down, give them abbreviated labels, and use them in this worksheet. (Please review the list in the book on page 102.)

These new labels are just for this worksheet. You do not need to memorize them (just the concepts behind them).

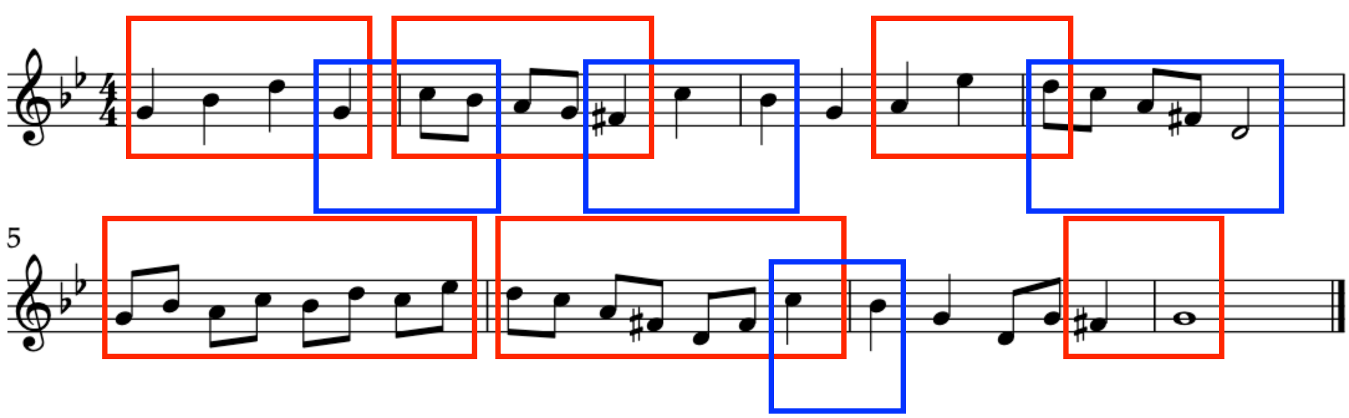
1. Step-wise movement = SW
   Outlining Harmony = OH

3. Leap / Step = LS

5. Diminished Intervals = DI

6. Active Tones
   - Leading Tone = LT
   - 7th of Chords = SC
   - Accidentals = AC

For the following musical excerpts, place the label (from the list above), that best fits the voice-leading in the boxed sections, <u>in the box</u> with the music. (See the examples) If a note has an X covering it, just consider it a non-harmonic tone and ignore it (for now).



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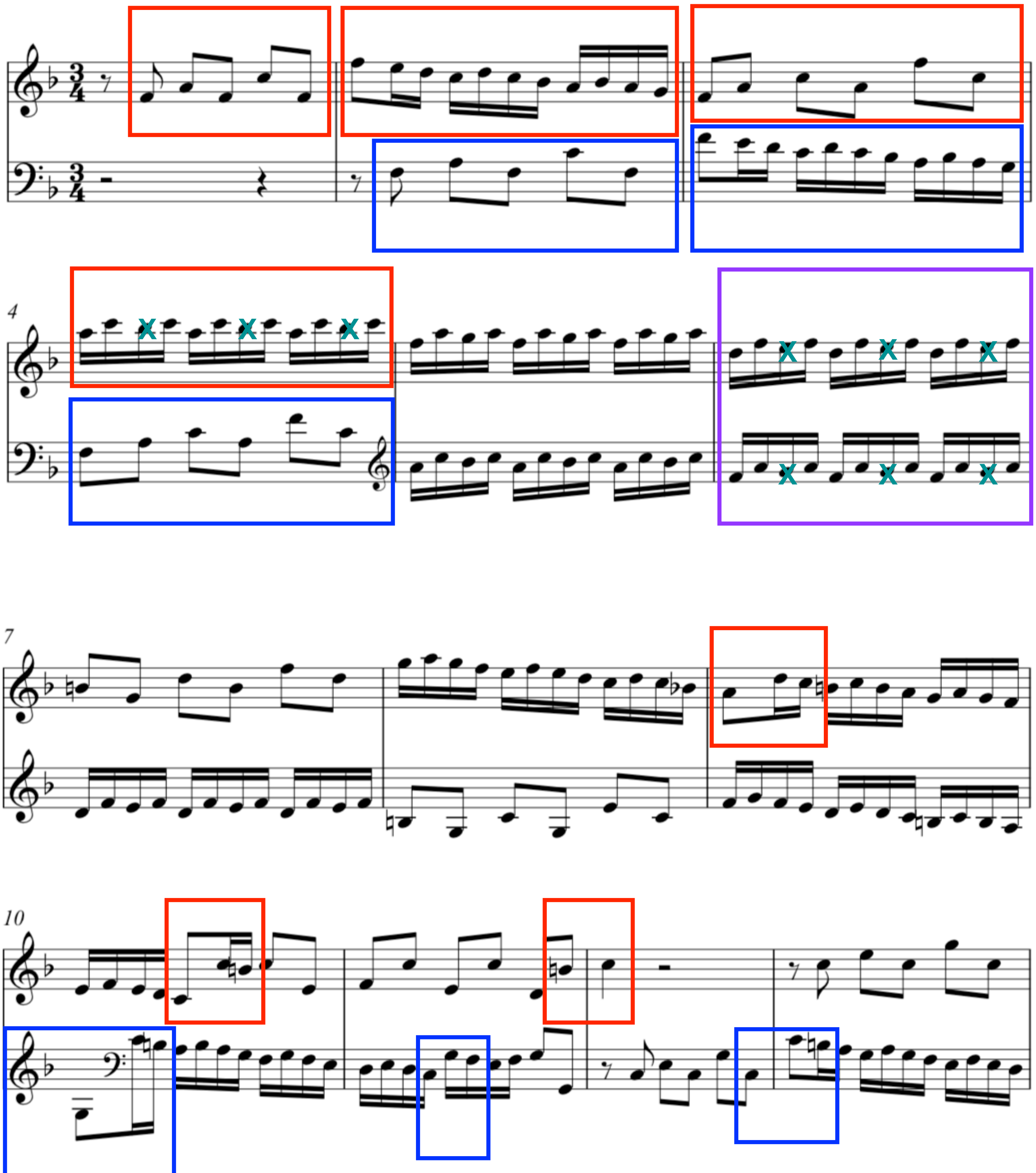
Voice-Leading - 2

What is the key for the previous selection? _____

In the previous selection, what is the cadence in measure 4? _____

In the previous selection, what is the harmony (Roman numeral) in measures 4 and 6? ____



Voice-Leading - 3

(For the previous selection)

What key does this piece begin in? _____

What is the chord (Roman numeral; no inversion) for measure 1? _____
(see the measure numbers at the beginning of each system)

What is the chord (Roman numeral; no inversion) for measure 6? _____

Starting in measure 10 there are no more B♭s.
What key is the music in at measure 12? _____

Analyze the last beat of measure 11 then measure 12. What cadence is that? _____

Canonic Conundrum

An Etude for Soprano and Tenor

Lon W. Chaffin

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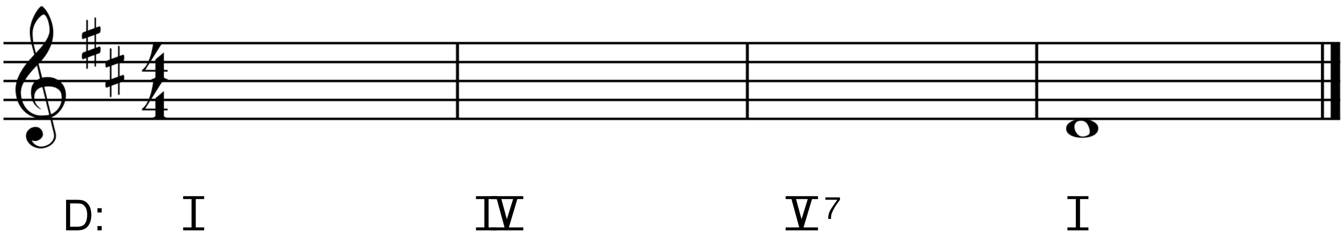
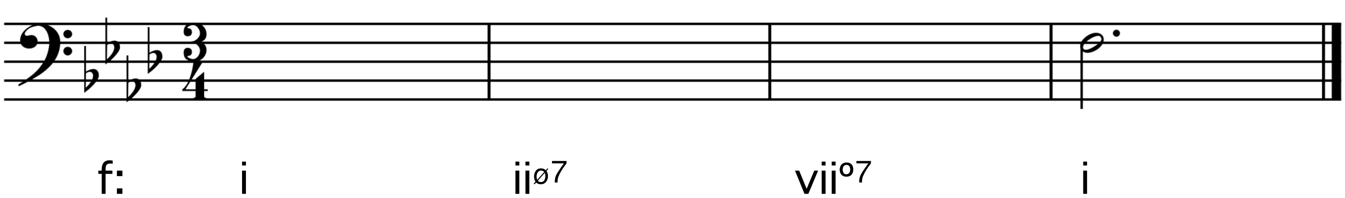
Voice-Leading - 4

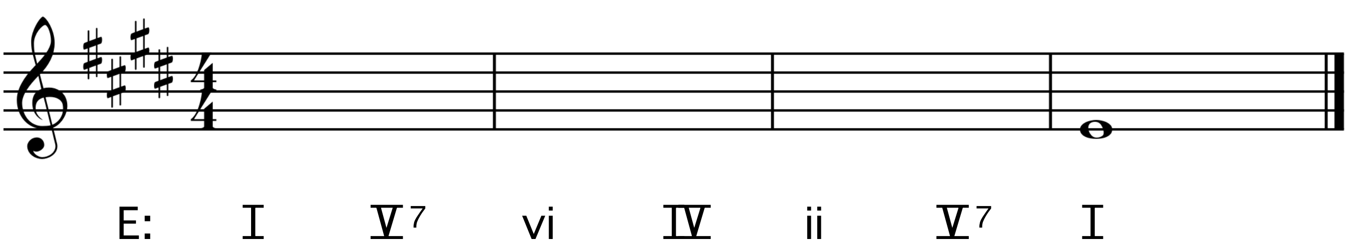
Within the parameters noted below, construct melodies following the guidelines provided on page 102 of the text.

- Use only quarter notes
- Make sure your melody fits within the meter and harmony provided
- Step-wise passing notes are acceptable between harmonic changes



(note the clef and the meter)





Note: The harmony in this last selection changes every two beats.

Part-Writing - 1

Name:________________________________ ID#___________

For the following hymn-tune (Darwall's 148th) we will be analyzing the part-writing. The following music is the tune with all four parts included, but we will be analyzing only two voices at a time.

We will break them down in this order: SA, ST, SB, AT, AB, TB

For the part-writing analysis, use the following designations:

- S = similar motion
- O = oblique motion
- C = contrary motion
- Pll5 = parallel 5ths
- Pll8 = parallel 8ves
- D5 = direct 5ths
- D8 = direct 8ves

From one set of vertical notes to the next, determine the type of movement. Put these designations (above) between the sets (see the examples below).

Part-Writing - 2

Soprano / Alto

Soprano / Tenor

Part-Writing - 3

Soprano / Bass

Alto / Tenor

Part-Writing - 4

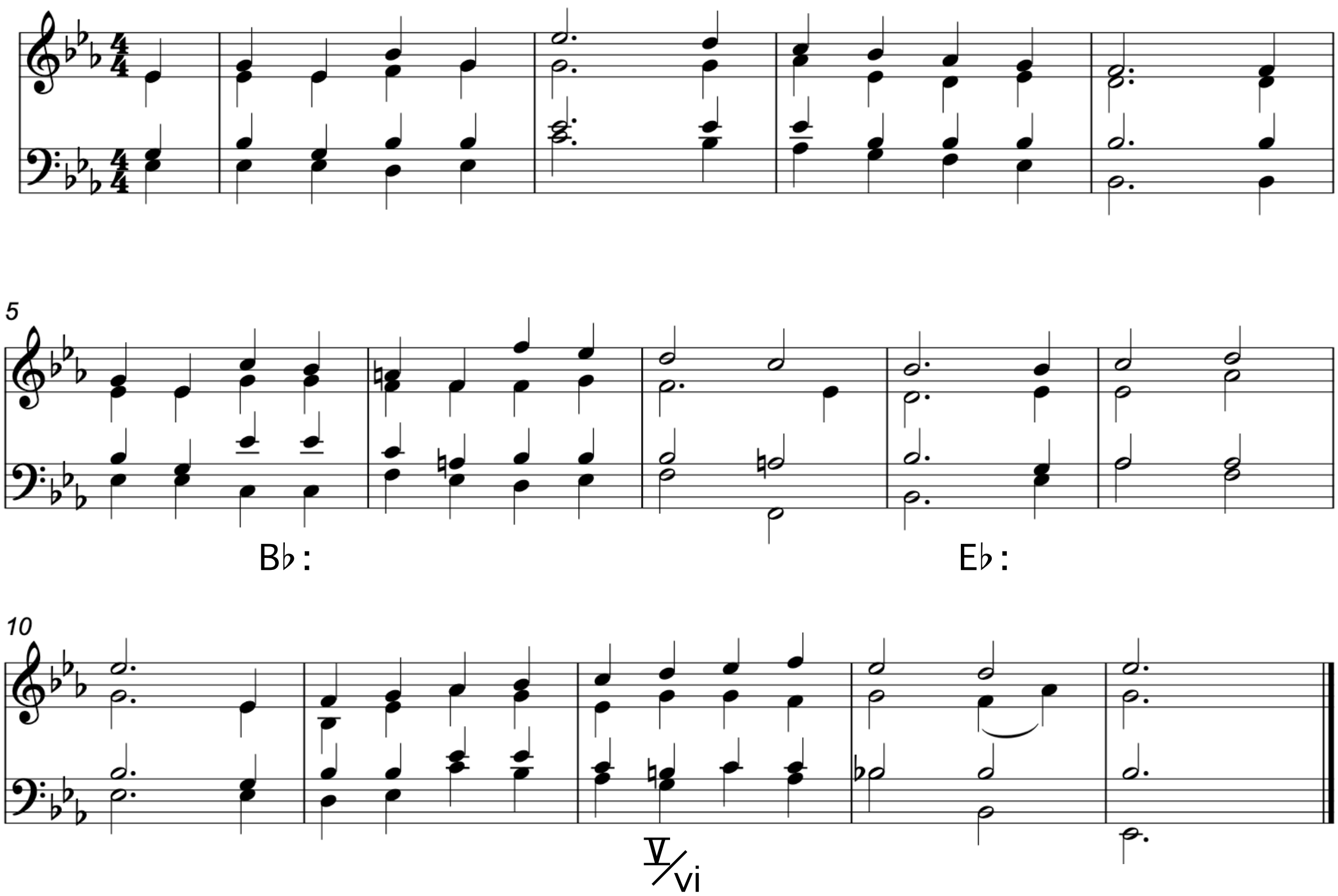
Alto / Bass

Tenor / Bass

Part-Writing - 5

Name:________________________________ ID#___________

For the following hymn-tune give a full harmonic analysis with Roman numerals and figured bass. Note the key changes in measures 5 and 8, and the advanced analysis in measure 12. For the key changes, simply analyze the music in the key indicated. For the symbols in measure 12, just give the appropriate analysis before and after them.



Part-Writing - 6

For each chord in this hymn-tune, indicate which, if any, chord members are doubled/ tripled or omitted. Use these labels:

-3 (3rd omitted)
-5 (5th omitted)
+R (root doubled or tripled)
+3 (3rd doubled)
+5 (5th doubled)
+7 (7th doubled)

Put the label for the omitted notes above the staff and the label for the doubled notes below the staff (see examples below).

If a chord has nothing doubled/tripled or omitted simply put nothing.

On the next page, using the labels from our previous worksheet, label each passage enclosed in a rectangle. (See examples)

Step-wise movement = SW
Outlining Harmony = OH
Leap / Step = LS
Diminished Intervals = DI

Active Tones
- Leading Tone = LT
- 7th of Chords = SC
- Accidentals = AC

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Part-Writing - 7

Also, circle any voice-leading movement that does not fall into our specific categories or is considered something to avoid.

For example:

- A leap not resolved appropriately
- An augmented interval
- An active tone not resolved appropriately

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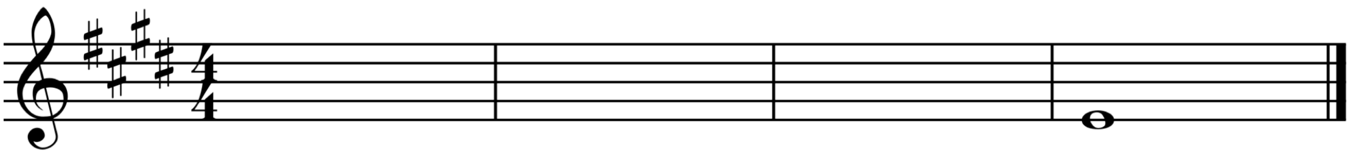
Melodic Construction

Name:_________________________________ ID#___________

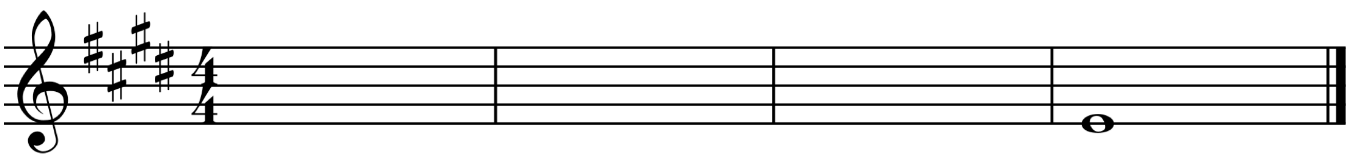
For the following harmonic progression, construct 3 different effective melodies, following our voice-leading guidelines.

Note: The harmony changes every two beats, on the 1st and 3rd beats of each measure.

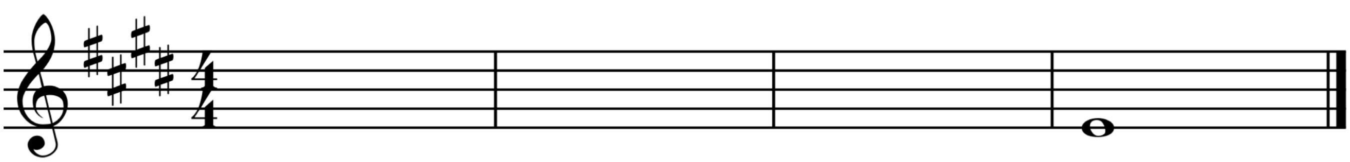
You may use quarter notes and eighth notes as your rhythmic material.



E: I V7 vi IV ii V7 I



E: I V7 vi IV ii V7 I



E: I V7 vi IV ii V7 I

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Closely-related Keys - 1

Name:________________________________ ID#___________

List every key that is closely-related to the keys given below. Use a single uppercase letter for major keys and a single lowercase letter for minor keys.

1. G Major _____ _____ _____ _____ _____
2. C Major _____ _____ _____ _____ _____
3. F Major _____ _____ _____ _____ _____
4. B♭ Major _____ _____ _____ _____ _____
5. E♭ Major _____ _____ _____ _____ _____
6. A♭ Major _____ _____ _____ _____ _____
7. D♭ Major _____ _____ _____ _____ _____
8. G♭ Major _____ _____ _____ _____ _____
9. C♭ Major _____ _____ _____ _____ _____
10. D Major _____ _____ _____ _____ _____
11. A Major _____ _____ _____ _____ _____
12. E Major _____ _____ _____ _____ _____
13. B Major _____ _____ _____ _____ _____
14. F♯ Major _____ _____ _____ _____ _____
15. C♯ Major _____ _____ _____ _____ _____
16. E Minor _____ _____ _____ _____ _____

Closely-related Keys - 2

17. A Minor ______ ______ ______ ______ ______

18. D Minor ______ ______ ______ ______ ______

19. G Minor ______ ______ ______ ______ ______

20. C Minor ______ ______ ______ ______ ______

21. F Minor ______ ______ ______ ______ ______

22. B♭ Minor ______ ______ ______ ______ ______

23. E♭ Minor ______ ______ ______ ______ ______

24. A♭ Minor ______ ______ ______ ______ ______

25. B Minor ______ ______ ______ ______ ______

26. F♯ Minor ______ ______ ______ ______ ______

27. C♯ Minor ______ ______ ______ ______ ______

28. G♯ Minor ______ ______ ______ ______ ______

29. D♯ Minor ______ ______ ______ ______ ______

30. A♯ Minor ______ ______ ______ ______ ______

Endnotes

1. Helmholtz, *Vorträgeund Reden*, 82

2. Hutchinson, “Celebrating,” 6

3. Pink, *Whole New Mind*, 139

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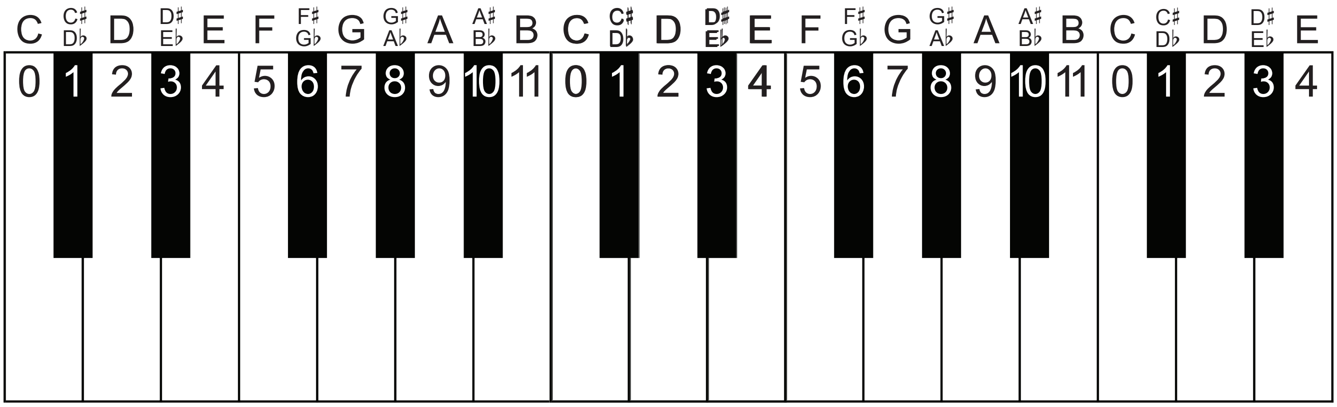
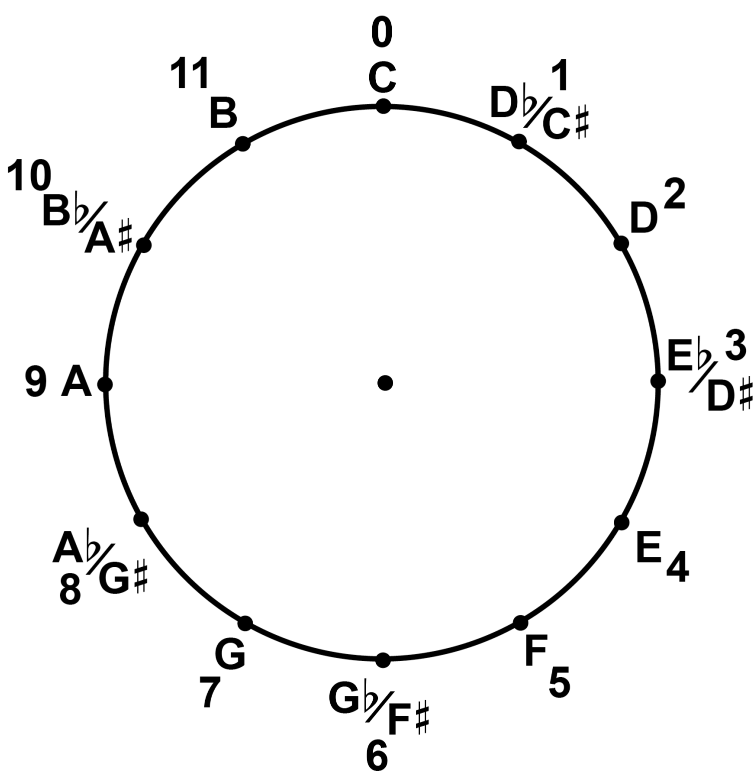
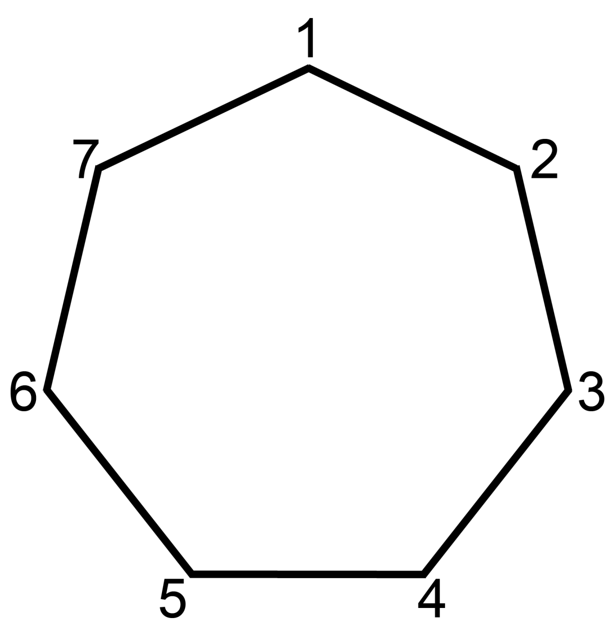
Graphics for Reference







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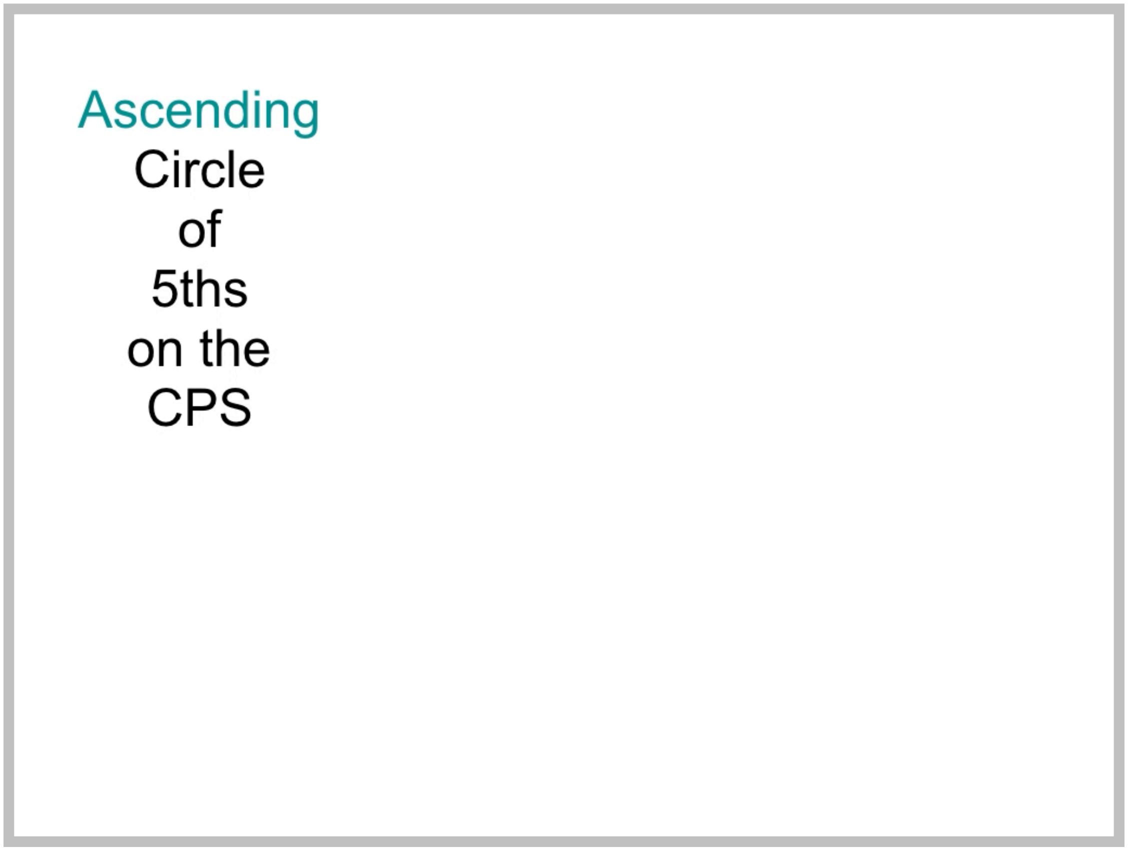
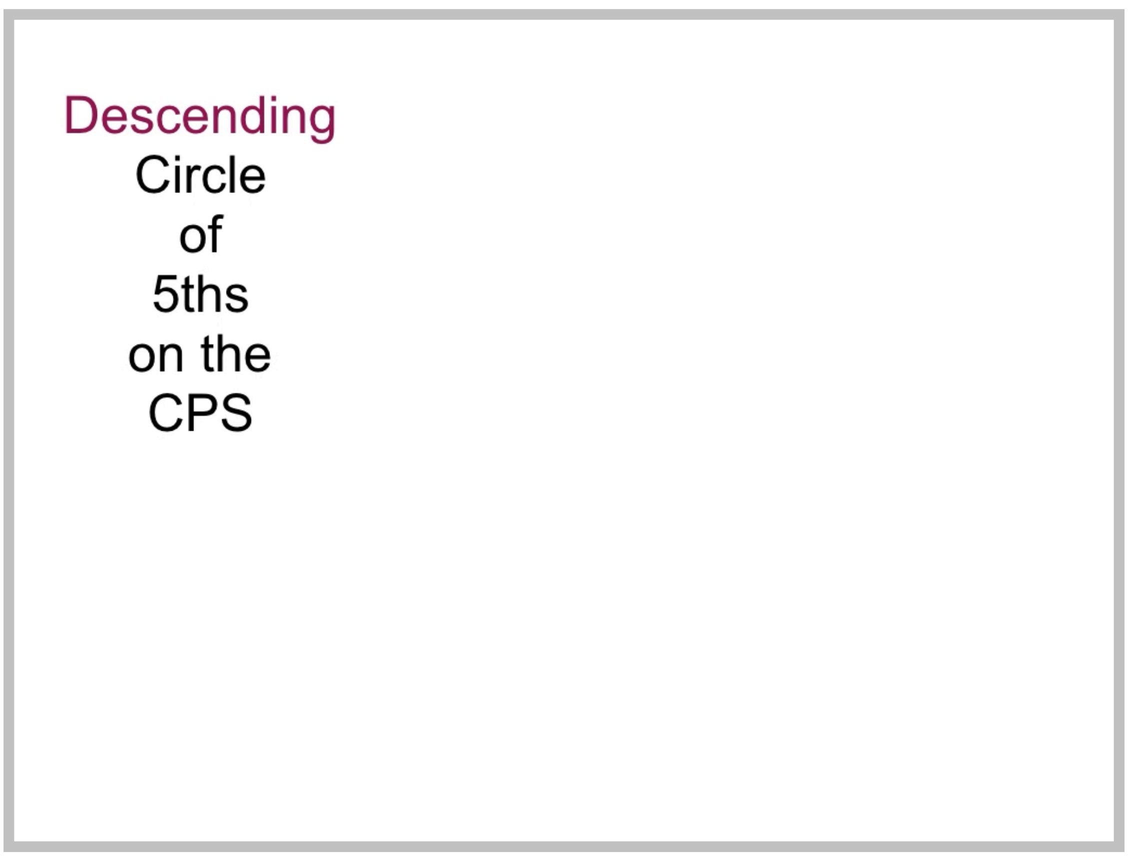
Circle of Fifths (these videos are only available in the ebook version)





STRONG ROOT MOVEMENT
CALCULATOR

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Math Equations and Charts

(only available in the PDF and ebook versions)

Click on the links below to take you to the noted information

Additional Math Connections

Mathematical Angles of Intervals and Inversions

The Geometry of Intervals

(only available in the ebook version)

Interval / Inversion Equations Chart

p = any pitch from the Chromatic Pitch Set

The chart can be read from left to right or from right to left

The number of half-steps in the inverted interval equations is found by subtracting the number of half-steps in the original equation from 12 (the inversion is the remainder of the octave)

Interval (Ascending)	Inverted Interval (Descending)
m2↑ = {p,(p + 1)} (mod 12)	M7↓ = {p,(p - 11)} (mod 12)
M2↑ = {p,(p + 2)} (mod 12)	m7↓ = {p,(p - 10)} (mod 12)
m3↑ = {p,(p + 3)} (mod 12)	M6↓ = {p,(p - 9)} (mod 12)
M3↑ = {p,(p + 4)} (mod 12)	m6↓ = {p,(p - 8)} (mod 12)
P4↑ = {p,(p + 5)} (mod 12)	P5↓ = {p,(p - 7)} (mod 12)
A4↑ = {p,(p + 6)} (mod 12)	d5↓ = {p,(p - 6)} (mod 12)
d5↑ = {p,(p + 6)} (mod 12)	A4↓ = {p,(p - 6)} (mod 12)
P5↑ = {p,(p + 7)} (mod 12)	P4↓ = {p,(p - 5)} (mod 12)
m6↑ = {p,(p + 8)} (mod 12)	M3↓ = {p,(p - 4)} (mod 12)
M6↑ = {p,(p + 9)} (mod 12)	m3↓ = {p,(p - 3)} (mod 12)
m7↑ = {p,(p + 10)} (mod 12)	M2↓ = {p,(p - 2)} (mod 12)
M7↑ = {p,(p + 11)} (mod 12)	m2↓ = {p,(p - 1)} (mod 12)

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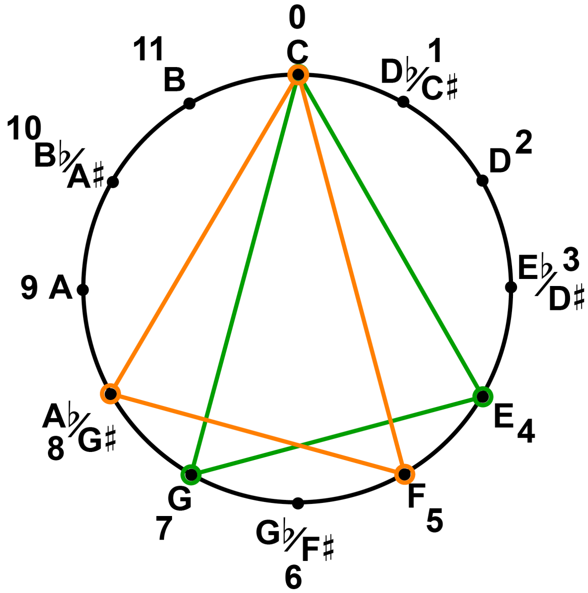
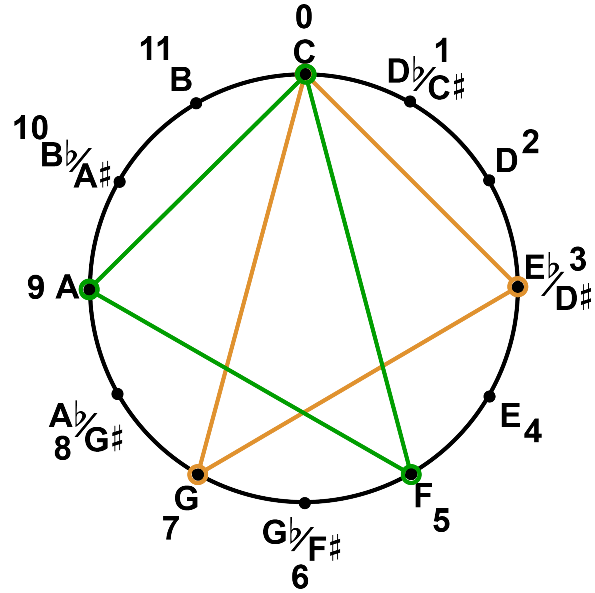
The Geometry of Triads and Seventh Chords

Examples of Complementary (Mirrored) Triads and Seventh Chords

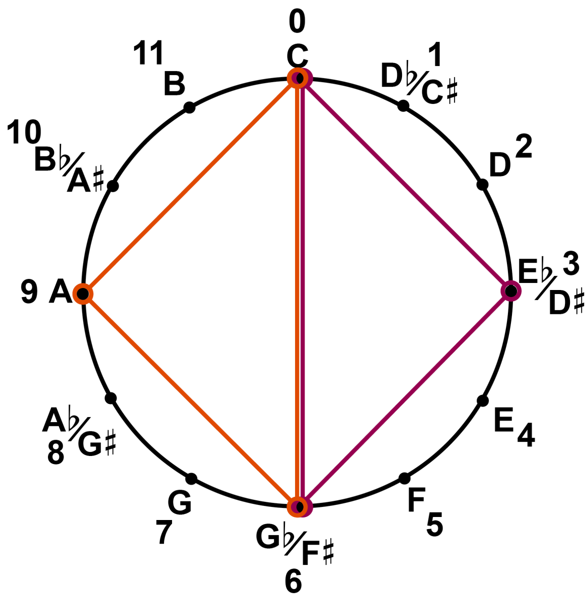
F minor ↔ C major



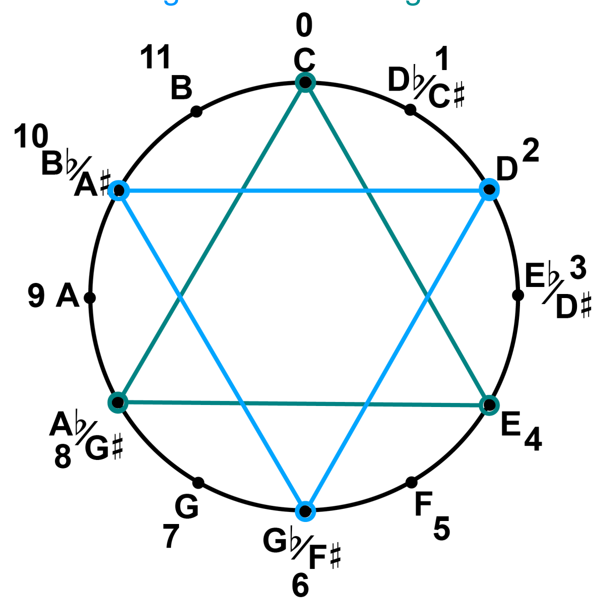
F major ↔ C minor



F♯ diminished ↔ C diminished



G♭ augmented ↔ C augmented



The purpose of including these graphs is not to just provide colorful geometric designs, but to demonstrate the connections between seemingly unrelated *triads* and *seventh chords*. Until we arrive at the study of *set theory* in our journey, let's just make note that the *chords* represented in each graph have the same combination of *intervals*. Those *intervals* may not be in the same sequence within the *chord* or start from the same *root*, but exist within both *chords*. In our graphs, for example:

F minor contains (m3, M3, P4)   C major contains (M3, m3, P4)

They have the same group of *intervals*, even though they have different *roots* and a different sequence for the *intervals*.

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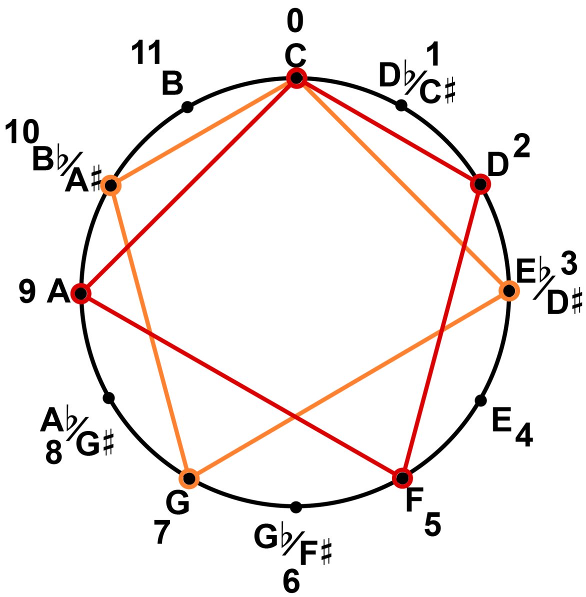
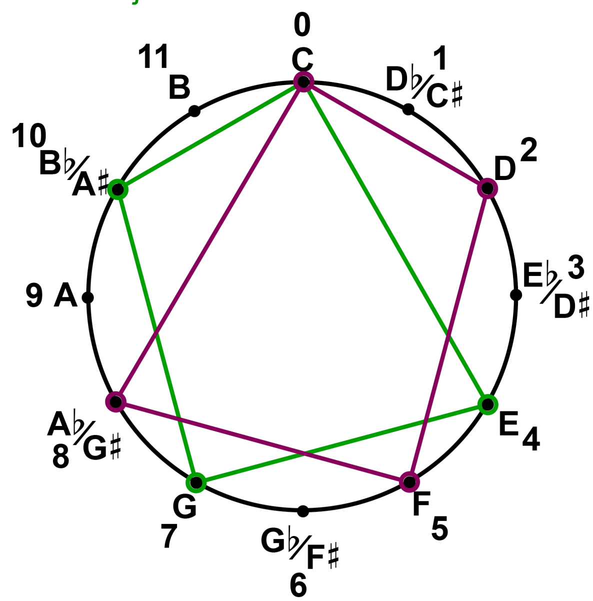
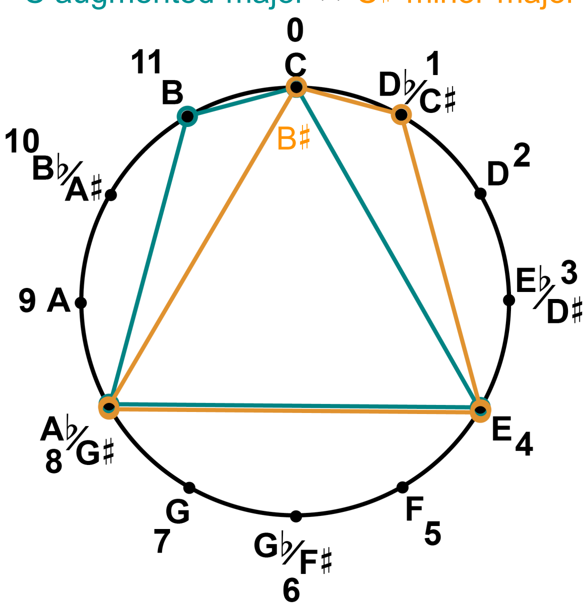
C minor-minor ↔ D minor-minor



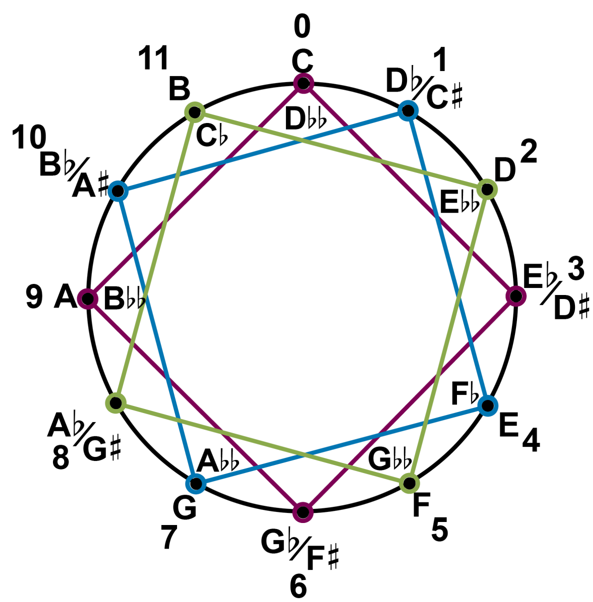
C major-minor ↔ D diminished-minor



C augmented-major ↔ C♯ minor-major



dd7 ↔ dd7 ↔ dd7 *



* With *enharmonic* respellings, there are 12 different dd7 *chords* in this graph

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